Non-dominated Sorting Based Fireworks Algorithm for Multi-objective Optimization

Mingze Li and Ying Tan(✉)

Key Laboratory of Machine Perception (MOE), School of Artificial Intelligence, Institute for Artificial Intelligence, Peking University, Beijing, China
{mingzeli,ytan}@pku.edu.cn

Abstract. Multi-objective optimization is one of the most important problem in the mathematical optimization. Some researchers have already proposed several multi-objective fireworks algorithms, of which S-metric based multi-objective fireworks algorithm (S-MOFWA) is the most representative work. S-MOFWA takes the hypervolume as the evaluation criterion of external archive updating, which is easy to implement but ignores the landscape information of the population. In this paper, a novel multi-objective fireworks algorithm named non-dominated sorting based fireworks algorithm (NSFWA) is proposed. The proposed algorithm updates the external archive with the selection operator based on the fast non-dominated sorting approach, which is specially designed for the spark generation characteristic of FWA to improve the diversity. A multi-objective guided mutation operator is also designed to enhance the efficiency of population information utilization and improve the search capability of the algorithm. Experimental results on the benchmarks demonstrate that NSFWA outperforms other multi-objective swarm intelligence algorithms of S-MOFWA, NSGA-II and SPEA2.

Keywords: Fireworks algorithm · Multi-objective optimization · Swarm intelligence · Non-dominated sorting based fireworks algorithm

1 Introduction

Fireworks algorithm (FWA) proposed by Tan et al. in 2010 is a novel swarm intelligence algorithm [14]. FWA has a double-layer structure, in which the higher layer is the global coordination between the firework populations and the lower one is the local search of a certain population. This hierarchical structure ensures that FWA can solve kinds of optimization problems with different landscape and shows a significant performance on the single-objective optimization problem. In recent years, guided fireworks algorithm (GFWA) [9], loser-out tournament fireworks algorithm (LoTFWA) [7] and other new variants [2,8,10,11,17,18] further

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enhance the performance of FWA from the aspects of global coordination and local exploitation.

Multi-objective optimization problem (MOP) is a kind of mathematical optimization problem with multiple conflicting objective functions to be optimized at the same time. Multi-objective optimization algorithm is aimed to find an optimal solution set composed of Pareto optimal solutions, which covering the whole Pareto front as completely as possible. Naturally, convergence and diversity are two main measures for MOP. Convergence mainly indicates the distance between the Pareto front and solutions obtained by the algorithm. And diversity can be roughly regarded as a ratio of the section covered by the solution set to the entire Pareto front.

According to the method of solution set updating, multi-objective optimization algorithms could be classified as two mainstream categories. Pareto dominance based methods such as non-dominated sorting genetic algorithm-II (NSGA-II) [4] and the improved strength Pareto evolutionary algorithm (SPEA2) [20] calculate the Pareto dominance between individuals in each iteration and update the solution set accordingly. Hypervolume indicator based methods like SMS-EMOA [1] use the volume covered by individuals instead of the Pareto dominance as the criterion to update the solution set.

Some researchers also proposed multi-objective FWA, and one of the most representative work is S-MOFWA proposed by Liu and Tan [12]. S-MOFWA adopted the hypervolume based framework and designed a novel external archive updating methods. The framework reduces the difficulty of multi-objective optimization significantly and makes it possible to inherit mechanisms of single-objective FWA. However, due to the limitation of the framework, S-MOFWA also ignores the information of dominated solutions and has a relatively low information utilization efficiency.

In this paper, a novel multi-objective FWA named non-dominated sorting based fireworks algorithm is proposed. NSFWA adopts a non-dominated sorting based external archive updating methods as the selection operator, and extends the idea of GFWA to MOP. In order to accelerate the convergence of MOFWA without affecting diversity, the multi-objective guided mutation operator is designed to generate guiding sparks with two different methods according to certain characteristic of fireworks. The adaptive amplitude mechanism and mapping rule are also revised.

The remaining parts is organized as follows. Some related works are introduced in Sect. 2. Our proposed algorithm is described in detail and the improved mutation operator is analyzed in Sect. 3. Then Sect. 4 presents the experimental results to present the good performance of NSFWA. Section 5 gives the conclusion.

2 Related Works

NSGA-II is one of the most influential multi-objective swarm intelligence algorithm. Swarm intelligence algorithms usually have a large number of populations and individuals, and thus it is necessary to calculate the dominance between individuals efficiently in MOP. Deb et al. proposed a fast non-dominated sorting algorithm in NSGA-II. The algorithm divides the population into several disjoint fronts $\{F_1, F_2, ..., F_m\}$ with the acceptable time complexity, and these fronts satisfies the dominance relation $F_1 \succ F_2 \succ ... \succ F_m$. Then the external archive or solution set could be updated accordingly. In order to keep the diversity of solutions, NSGA-II also introduced a density indicator named crowding distance as the other updating criterion. The fast non-dominated sorting algorithm provides efficient evaluation and updating framework for many algorithms. However, directly applying them on FWA would obtain a solution set with lower diversity.

Liu et al. adopted another mainstream framework in S-MOFWA. S-MOFWA update the external archive according to the S-metric which is a kind of hypervolumes indicator. Intuitively, S-metric could be regarded as the space that only dominated by a certain solution in the entire solution set, and the solution with better S-metric usually locates in the area with lower density and closer to the Pareto front. Thus, S-metric could unify convergence measure and diversity measure into one indicator, and simplify the framework of multi-objective swarm intelligence algorithm. Whereas, the calculation method of S-metric in S-MOFWA is only applicable for the non-dominated solutions, and the S-metrics of dominated solutions are assigned as 0. This characteristic leads to the lost of population information and reduce the information utilization efficiency.

Based on the previous works, this paper redesigns the operators in FWA, and proposes a Non-dominated Sorting Based Fireworks Algorithm with higher information utilization ratio.

3 Non-dominated Sorting Based Fireworks Algorithm

3.1 Framework

NSFWA is mainly composed of explosion operator, non-dominated sorting based selection operator, multi-objective guided mutation operator, mapping rule and adaptive explosion amplitude mechanism, and its principle to improve the convergence and diversity of the algorithm with the population information.

Initialization. The initialization of NSFWA is same as the single-objective FWA. NSFWA generates N fireworks randomly in the decision space D:

$$\mathbf{x}_i = (x_{i1}, x_{i2}, ..., x_{in}),\ i = 1, 2, ..., N, \tag{1}$$

where n is the dimension of decision space.

Explosion Operator. The explosion operator of NSFWA also follows the single-objective FWA and randomly generates a certain number of explosion sparks in the hyperspace with firework $\mathbf{x}_i$ as the center and explosion amplitude

A_i as the radius. If the generated explosion spark is out of the bound, it would be remapped into the feasible region according to a certain rule. The mapping rule used in NSFWA is the midpoint mapping, and it would be introduced in the following section.

Mapping Rule. Traditional mapping rule is random mapping, that is, if some dimensions of the spark is out of the bound, the values of the corresponding dimensions would be randomly generated again until the spark is completely within the feasible region. The explosion amplitude is usually decreased during the search process, thus, FWA is tend to have a poor performance on the problem that the global optimum locates near the bound. And the random mapping would exacerbate the problem sometimes. Shown as the Algorithm 1, midpoint mapping rule would reset the dimension that out of the bound as the midpoint of the bound and firework.

Algorithm 1. Midpoint Mapping Rule

Input: Firework $\mathbf{x}_{ij}$, explosion spark $\mathbf{s}_{ij}$, upper bound ub, lower bound lb
Output: Explosion spark $\mathbf{s}_{ij}$

for $k = 1$ to n **do**
  if $s_{ij}^{(k)} > ub$ **then**
    $s_{ij}^{(k)} \leftarrow \frac{1}{2}(x_{ij}^{(k)} + ub)$
  end if
  if $s_{ij}^{(k)} < lb$ **then**
    $s_{ij}^{(k)} \leftarrow \frac{1}{2}(x_{ij}^{(k)} + lb)$
  end if
end for
return explosion spark $\mathbf{s}_{ij}$

Compared with other mapping rules, midpoint mapping could help the population find the optimum near the bound of feasible region, and ensure that population also has the ability to escape from the bound.

Selection Operator. Non-dominated sorting based selection operator is used to update the external archive and select new fireworks. In NSFWA, fireworks, explosion sparks and the individuals in the external archive compose the candidate pool, and the selection operator would select N_R individuals from the candidate pool in to the external archive of the next generation, where the top N individuals would be identified as the new fireworks. Concretely, the operator divides the candidate pool C into several disjoint fronts $\{F_1, F_2, ..., F_m\}$ according to the fast non-dominated sorting algorithm proposed by Deb, and these fronts satisfied the definition and dominance listed as the following:

$$F_k = \{\mathbf{x} | n_{\mathbf{x}} = k, \mathbf{x} \in C\}, \tag{2}$$

$$F_1 \succ F_2 \succ ... \succ F_m, \tag{3}$$

where $n_{\mathbf{x}}$ is the number of individuals that dominate $\mathbf{x}$. Then, the candidate solutions would be put into the external archive from front F_1 successively, until a certain front F_k cannot be entirely put in. In order to determine which candidate solutions in F_k are supposed to be put into archive, NSGA-II introduces the crowding distance as the indicator. For solution $\mathbf{x}_m$ in front F_k, its crowding distance can be defined as the following:

$$D(\mathbf{x}_m) = \sum_{i=1}^{r} |f_i(\mathbf{x}_{m+1}) - f_i(\mathbf{x}_{m-1})|, \tag{4}$$

where $\mathbf{x}_{m-1}$ and $\mathbf{x}_{m+1}$ are the neighbors of $\mathbf{x}_m$, and r is the number of objective function. Fig. 1 shows the calculation of the crowding distance.

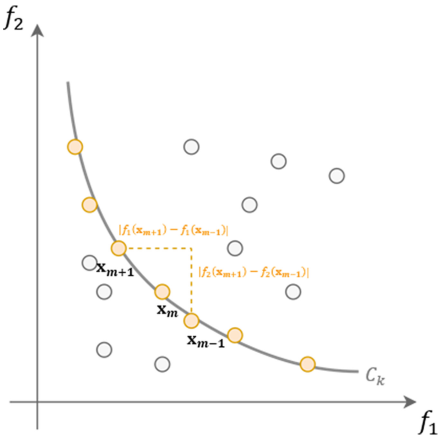


Fig. 1. The calculation of crowding distance.

The solutions with larger crowding distance tend to locate in a low density area, and would be selected into the archive. As mentioned above, explosion sparks are generated in a specific hyperspace centered on fireworks, which means that the diversity of the entire solution group is highly related to the density of fireworks' location. Therefore, NSFWA not only sorts the front P_k but also the first front P_1 to ensure diversity of fireworks. The entire process of selection operator is shown as the Algorithm 2.

Adaptive Amplitude Mechanism. Explosion amplitude is one the decisive factors of the global exploration and local exploitation. In NSFWA, amplitude is adjusted adaptively according to the dominance relation between the current fireworks and the previous fireworks in each generation. The parameter setting

Algorithm 2. Selection Operator

Input: Fireworks $X_t = \{\mathbf{x}_1, \mathbf{x}_2, ...\mathbf{x}_N\}$, sparks $S_t = \{S_{1,t}, S_{2,t}, ..., S_{N,t}\}$, external archive R_t, size of archive N_R

Output: external archive R_{t+1}, fireworks X_{t+1}, sorted candidate pool $C_{sorted,t}$

Calculate the fitness values of X_t and S_t
Generate $C_t = \{X_t \cup S_t \cup R_t\}$
Sort the candidate pool $P = \text{FastNonDominatedSorting}(C_t), P = \{P_1, P_2, ..., P_m\}$
Declare the external archive $R_{t+1} = \emptyset$ and the counter $i = 1$
while $|R_{t+1}| + |P_i| \leq N_R$ **do**
  if i=1 **then**
    Calculate the crowding distance D_i of individuals in front P_i
    Sort the individuals in P_i by the descending order of crowding distance
  end if
  $R_{t+1} = R_{t+1} \cup P_i$
  $i = i + 1$
end while
Calculate the crowding distance D_i of individuals in front P_i
Sort the individuals in P_i by the descending order of crowding distance
$R_{t+1} = R_{t+1} \cup P_i[1 : N_R - |R_{t+1}|]$
$X_{t+1} = R_{t+1}[1 : N]$
$C_{sorted,t} = P_1 \cup P_2 \cup ... \cup P_m$
return R_{t+1}, X_{t+1}, $C_{sorted,t}$

of the mechanism refers to the one-fifth success rule proposed by Schumer and Steiglitz [13], and adopt a simple implementation of it [6]:

$$A_{i,t+1} = A_{i,t} \cdot \begin{cases} \alpha & if\ \mathbf{x}_{i,t+1} \succ \mathbf{x}_{i,t}\ and\ A_{i,t} \cdot \alpha \leq ub - lb \\ \alpha^{-\frac{1}{4}} & if\ \mathbf{x}_{i,t+1} \preceq \mathbf{x}_{i,t}\ and\ A_{i,t} \cdot \alpha \geq \beta \cdot (ub - lb), \end{cases} \tag{5}$$

where α is a hyper parameter controlling the change rate of amplitude, and β is a hyper parameter used to set the minimum of amplitude. In the early phase of search, there is a relatively high probability for population to find a better solution, and the amplitude tends to increase. On the contrary, it is more difficult for the population to make progress, and the amplitude tends to decrease. Therefore, the amplitude usually changes from large to small, which means that the population is encouraged to explore globally in the early phase and exploit a certain local area in the later.

Mutation Operator. In order to further enhance the local search capability of the algorithm by using the population information, NSFWA designs a novel operator named multi-objective guided mutation operator. Different from the single-objective FWA, the guided mutation operator in MOP must be executed after the selection operator obtaining the fitness information of populations. The main idea of the mutation operator is to calculate the difference between solutions and the solutions dominated by them, and add the difference on the location of fireworks to generate the mutation sparks. These mutation sparks usually has a better fitness compared with the fireworks. The entire process is shown as the Algorithm 3.

Algorithm 3. Multi-objective Guided Mutation Operator

Input: Firework $\mathbf{x}_{i,t+1}$, sorted candidate pool $C_{sorted,t} = \{\mathbf{c}_1, \mathbf{c}_2, ..., \mathbf{c}_{|C|}\}$, group ratio σ, group size μ
Output: Guided mutation spark $\mathbf{g}_{i,t+1}$

```
if Firework x_{i,t+1} is not from the external archive then
    Extract the population of x_i from C_{sorted,t} and keep the relative order
    Calculate the guiding vector Δ_i = 1/(σλ_i) (Σ_{j=1}^{σλ_i} s_ij − Σ_{j=λ_i−σλ_i+1}^{λ_i} s_ij)
end if
if Firework x_i is from the external archive then
    Δ_i = 1/μ (Σ_{j=1}^{μ} c_{rand(0,σ|C|)} − Σ_{j=1}^{μ} c_{rand(σ|C|−μ+1,σ|C|)})
end if
Generate the explosion spark g_i = x_i + Δ_i
return g_i
```

Step 3: $\Delta_i = \frac{1}{\sigma\lambda_i}\left(\sum_{j=1}^{\sigma\lambda_i} \mathbf{s}_{ij} - \sum_{j=\lambda_i-\sigma\lambda_i+1}^{\lambda_i} \mathbf{s}_{ij}\right)$

Step 6: $\Delta_i = \frac{1}{\mu}\left(\sum_{j=1}^{\mu} \mathbf{c}_{rand(0,\sigma|C|)} - \sum_{j=1}^{\mu} \mathbf{c}_{rand(\sigma|C|-\mu+1,\sigma|C|)}\right)$

To accelerate the convergence, the mutation operator directly use new firework selected by the selection operator to calculate the guiding spark (GS). However, the firework might come from the external archive and have already lost its population, thus the mutation calculate the guiding vector (GV) with two different methods:

1. If the firework $\mathbf{x}_{i,t+1}$ is not from the external archive: the firework have its own population in the candidate pool, and the population is already sorted after the selection operator. The guiding vector would be calculated as the difference between the centroid of the top $\sigma\lambda_i$ sparks and the bottom $\sigma\lambda_i$ sparks in the population.
2. If the firework $\mathbf{x}_{i,t+1}$ is from external archive: the operator calculate the difference between two groups that formed by μ solutions in the top $\sigma|C|$ solutions and μ solutions in the bottom $\sigma|C|$ solutions respectively to generate the guiding vector. Here, the μ solutions in two groups are selected randomly, so as to avoid that fireworks share a same guiding vector and affect the diversity of the final solution set.

Guiding sparks would be put into the external archive and participate the selection in the next generation, but not replace fireworks directly.

The framework of the Non-dominated Sorting Based Fireworks Algorithm is shown as the Algorithm 4.

3.2 Principle and Analysis

Analysis of Multi-objective Guided Mutation Operator. The main purpose of multi-objective guided muation opeartor is to improve the search capability of NSFWA without affecting the diversity of the solution set.

Algorithm 4. Non-dominated Sorting Based Fireworks Algorithm

Input: upper bound ub, lower bound lb, number of firework$N$, number of spark λ, external archive size N_R, group ratio σ, group size μ, explosion amplitude A, change rate α, minimum parameter β
Output: Optimal solution set

```
Initialize N fireworks in the decision space D bounded by lb and ub
Declare the external archive R = {X_1} and generation counter t = 1
while termination condition not met do
    Generate explosion sparks S_t = ExplosionOperator(X_t, A_t, λ_t)
    Update external archive, candidate pool and fireworks R_{t+1}, C_{sorted,t}, X_{t+1} =
        SelectionOperator(X_t, S_t, R_t)
    Adjust explosion amplitude A_{t+1} = AdaptiveAmplitude(X_t, X_{t+1}, A_t, α, β)
    Generate mutation sparks G_t = MultiObjectiveMutationOperator(X_t, C_{sorted,t},
        σ, μ)
    Update external archive R_{t+1} = R_{t+1} ∪ G_t
    t = t + 1
end while
return {R_t \ G_{t-1}}
```

Different with the single-objective optimization, the multi-objective optimization algorithm searches for a entire Pareto front composed of several Pareto optimums rather than a certain global optimum. Therefore, all directions that make the new individual generated on the direction better than the original fireworks are acceptable related directions. The following visualization will explain why the guiding spark could guide the population to search along the relevant direction.

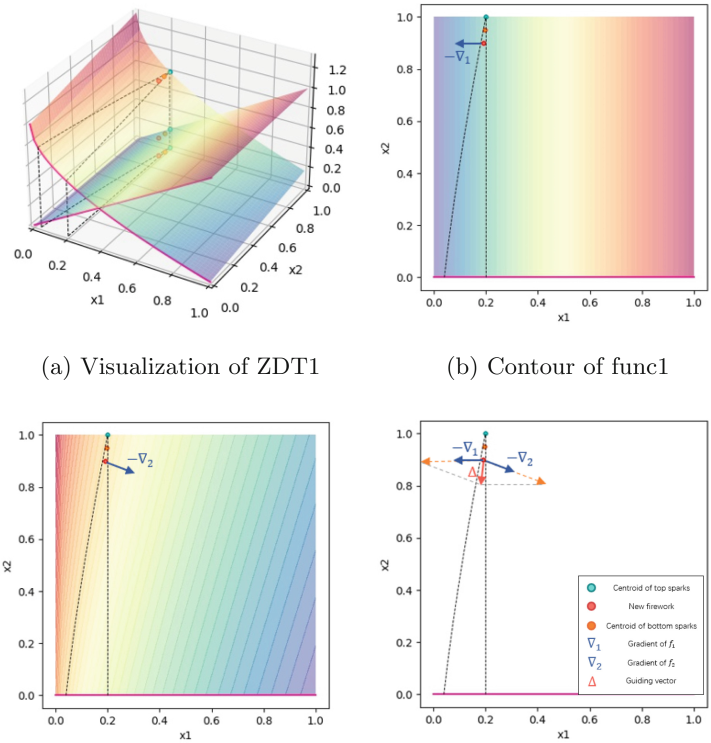
Suppose the optimization problem is ZDT1 test function, which includes two convex objective functions. Set the dimension of decision vector as 30, and the Pareto optimal solutions of ZDT1 is 0 on all dimensions except x_1. The objective functions could be visualized on the first two dimensions as the Fig. 2a, where the pink segment is the projection of Pareto front on the objective function. And the guiding vector could be approximately decomposed as the weighted sum of negative gradients of two objective functions (See Fig. 2b–Fig. 2d):

$$\varDelta = w_1 \nabla_1 + w_2 \nabla_2,\ w_1 \leq 0 \ and\ w_2 \leq 0, \tag{6}$$

where ∇_1 and ∇_2 are gradients.

Thus, guiding spark is likely to obtain a better fitness value on at least one objective function compared with the firework, which means these non-dominated elite individuals might be selected as the firework in the next generation. Guided by the elite individuals, populations could approach the Pareto front stably and the search ability of NSFWA are also enhanced.

Selection of Parameters. Now, suppose there are two fireworks that selected from the external archive and locate in a same region. If they share a same guiding vector, then their population is tend to move towards a same region



(a) Visualization of ZDT1

(b) Contour of func1

(c) Contour of func2

(d) Decomposition of GV

Fig. 2. Principle of multi-objective guided mutation function.

of the Pareto front, and this would be harmful for the diversity. The random mechanism is introduced to alleviate the problem. Generally speaking, the larger group ratio σ and smaller group size μ means stronger randomness, and the diversity of the solution set could be better. On the contrary, the convergence could be accelerated but the diversity might be weakened. The analysis above could be a basis for selection of parameters.

4 Experiments

To illustrate the performance of NSFWA, experiments on several test functions were conducted, and S-MOFWA, NSGA-II, SPEA2 and RVEA [3] are selected as baselines. Besides, the ablation experiments were also conducted to verify the effectiveness of operators and mechanisms in NSFWA.

4.1 Experimental Setup

The test functions in this paper include Schaffer's problem (SCH) [16], Kursawe's problem(KUR) [16] and ZDT test functions [19].

For NSFWA, the number of firework $N = 10$, the total number of explosion spark $\lambda = 100$, the size of external archive $N_R = 100$, amplitude change rate $\alpha = 1.2$, minimum parameter $\beta = 0.2$. The group ratio σ and group size μ are set as 0.3 and 10 respectively for all benchmarks except ZDT2. σ and μ are set as 0.5 and 5 for ZDT2. For S-MOFWA, parameters are set as [12]. And other baseline algorithms refer to platform Geatpy [5]. The platform is Ubuntu 18.04 with Intel(R) Xeon(R) CPU E5-2675 v3. Each test function runs 20 times repeatedly with the maximal evaluation number of 200000.

4.2 Experimental Criterion

Generational distance (GD) [15] and hypervolume (HV) are adopted as the criteria to evaluate the diversity and convergence respectively in this paper.

Generational Distance. Generational distance can be regarded as the average of the minimal distance between solutions obtained and the theoretical Pareto front in objective space:

$$GD = \frac{\sqrt{\sum_{i=1}^{n} d_i^2}}{n}, \tag{7}$$

where d_i is the minimal distance between individual i and the theoretical front, and n is the size of solution set. 500 solutions are generated uniformly on the theoretical front of each test function as the reference for calculating GD except KUR problem. (100 solutions selected for KUR.)

Hypervolume. Hypervolume is one of the most applied criterion of MOP. HV is the volume of the objective space that covered by optimal solution set obtained:

$$S(M) = \Lambda(\cup_{i=1}^{n} \{\mathbf{x} | \mathbf{x}_i \succ \mathbf{x} \succ \mathbf{x}_{ref}\}), \tag{8}$$

where Λ represents Lebesgue measure, and $\mathbf{x}_{ref}$ is a reference point that dominated by all solutions. Actually, HV can not only evaluate the diversity but also the convergence. The reference point selected for SCH, KUR and ZDT1-6 are $(4, 4)$, $(-14, 1)$, and $(1, 1)$ respectively.

4.3 Experimental Results

Ablation Experiments. To verify the effectiveness of the selection operator, mutation operator and mapping rule proposed in this paper, ablation experiments take the following algorithm as comparison: (i) NSFWA - CD: NSFWA without crowding distance sorting of firework in selection operator, (ii) NSFWA - GS: NSFWA without multi-objective guided mutation operator and (iii) NSFWA

Table 1. Generational distance of ablation experiments.

	NSFWA		NSFWA - CD		NSFWA - GS		NSFWA + RM	
Func.	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SCH	3.27E−03	3.45E−04	**3.17E−03**	1.74E−04	3.16E−03	2.88E−04	3.29E−03	**1.10E−04**
KUR	5.10E−02	2.91E−03	**5.04E−02**	**1.39E−03**	5.46E−02	2.45E−03	5.02E−02	1.84E−03
ZDT1	**1.10E−03**	**3.69E−05**	1.21E−03	1.15E−04	8.22E−02	3.49E−03	8.10E−01	2.88E−02
ZDT2	8.03E−04	**3.42E−05**	**7.84E−04**	5.88E−05	1.41E−01	4.27E−03	1.44E−01	2.24E−03
ZDT3	**1.07E−03**	8.82E−05	1.19E−03	**5.21E−05**	4.72E−02	6.99E−04	8.25E−01	3.15E−02
ZDT6	**5.92E−04**	**2.32E−05**	7.43E−03	7.41E−04	1.24E+00	3.54E−01	1.48E+00	1.61E−01

Table 2. Hypervolume of ablation experiments.

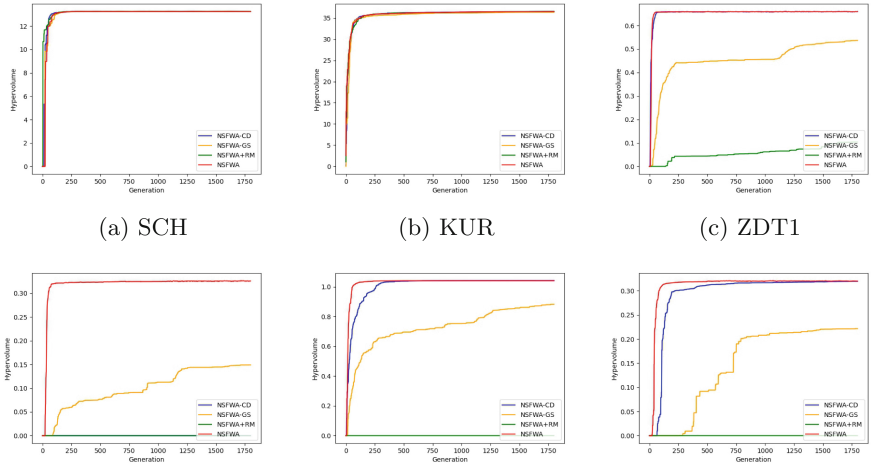
	NSFWA		NSFWA - CD		NSFWA - GS		NSFWA + RM	
Func	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SCH	1.33E+01	**3.79E−05**	**1.45E+01**	3.30E−03	1.32E+01	2.98E−03	1.32E+01	1.44E−03
KUR	**3.68E+01**	**4.03E−02**	3.66E+01	5.47E−02	3.65E+01	4.31E−02	3.68E+01	7.69E−02
ZDT1	**6.61E−01**	**3.24E−05**	6.59E−01	1.83E−04	5.48E−01	4.51E−03	1.23E−01	2.78E−02
ZDT2	**3.28E−01**	1.18E−04	3.26E−01	**9.61E−05**	1.68E−01	2.81E−03	1.67E−01	2.67E−03
ZDT3	**1.04E+00**	**9.71E−06**	1.04E+00	6.22E−05	8.91E−01	8.67E−03	3.51E−01	3.80E−02
ZDT6	**3.21E−01**	**7.93E−04**	3.18E−01	3.30E−03	2.20E−01	9.79E−02	1.81E−01	9.97E−02

+ RM: NSFWA using random mapping rule. The experimental results are shown as Table 1 and Table 2.

Complete NSFWA wins a better HV than NSFWA without crowding distance sorting, and it could be seen that sorting fireworks in selection operator could improve the diversity. The better GD indicates that guided mutation operator improves the search capability of NSFWA significantly. And the HV curve (See Fig. 3) also proves that the mutation operator could accelerate the convergence of NSFWA. NSFWA using midpoint mapping outperforms NSFWA using random mapping obviously on ZDT test functions. It is worthy noting that most of optimal solutions of ZDT locate near the lower bound, which means that midpoint mapping improve the performance of FWA on the kind of problem.

Comparison with Other Algorithms. Table 3 and Table 4 gives the results of NSFWA and other algorithms. The average rank of GD of NSFWA is 1.83 and the average rank of HV is 1.50, which is the best compared with other algorithms. The difference between the average rank of GD and HV indicates that NSFWA performs better on the diversity and could obtain a solution set covering larger target space.

Among the benchmarks, SCH and ZDT1 have convex Pareto fronts. The means and standard deviations on these two problems show that NSFWA has a stable and good performance on problem with a convex front. KUR and ZDT2 has non-convex Pareto fronts. The performance of NSFWA is slightly worse than S-MOFWA and NSGA-II on KUR, but better on ZDT2. As mentioned above, the setting of group ratio σ and group size μ for ZDT2 is different with other problems. It is inferred that populations is easily to be trapped in a certain part



(a) SCH (b) KUR (c) ZDT1

(d) ZDT2 (e) ZDT3 (f) ZDT6

Fig. 3. Hypervolume curve of ablation experiments.

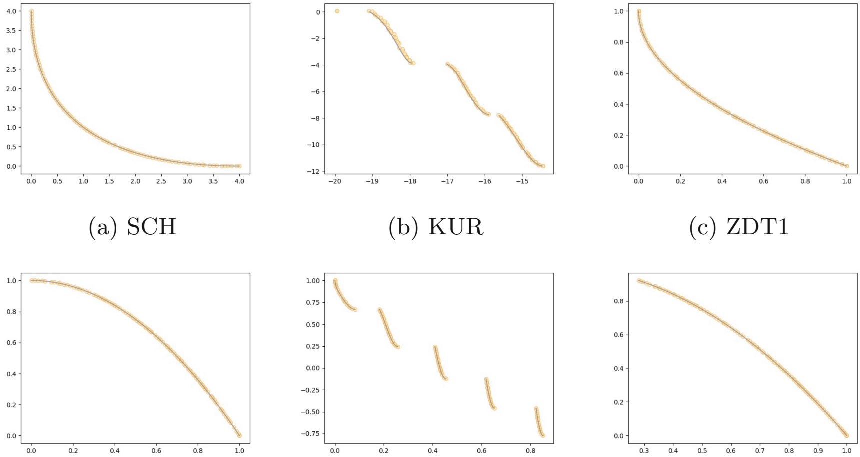
of global optimum in the test of ZDT2, and thus there is a need of stronger randomness to help populations get rid of the area. The Pareto front of ZDT3 is composed of several non-contiguous convex parts, which also requires better diversity of population, and NSFWA outperforms other algorithms on both GD and HV. The decision space and Pareto front of ZDT6 is non-uniform. The density of individuals is gradually lower when they locate closer to the Pareto front. NSFWA ranked second on both GD and HV with a small gap from the best. Generally speaking, the results indicate that NSFWA performs well on kinds of functions with different characteristics of Pareto front. The solution set obtained by NSFWA is visualized as Fig. 4.

Table 3. Generational distance of NSFWA and other algorithms.

	NSFWA		*S*-MOFWA		NSGA-II		SPEA2		RVEA	
Func.	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SCH	3.27E−03	3.45E−04	3.32E−03	9.62E−05	3.33E−03	**7.82E−05**	4.16E−03	4.27E−04	**3.03E−03**	1.83E−04
KUR	5.10E−02	2.91E−03	**3.57E−02**	1.83E−03	3.77E−02	3.06E−03	6.84E−01	**1.12E−03**	4.05E−02	8.25E−03
ZDT1	**1.10E−03**	**3.69E−05**	1.47E−03	4.37E−05	1.41E−03	8.55E−05	1.69E−03	1.90E−04	1.76E−03	3.48E−04
ZDT2	**8.03E−04**	3.42E−05	1.17E−03	6.22E−05	1.06E−03	1.54E−04	1.04E−03	**1.14E−05**	1.21E−03	2.06E−04
ZDT3	**1.07E−03**	8.82E−05	4.01E−03	1.88E−03	1.09E−03	**6.96E−05**	1.96E−01	1.95E−01	1.64E−03	1.15E−04
ZDT6	5.92E−04	2.32E−05	**5.66E−04**	**1.83E−05**	6.30E−04	9.65E−05	1.09E−01	6.87E−02	6.44E−04	1.86E−05
AR	1.83	2.50	2.67	2.33	2.67	2.83	4.33	3.50	3.50	3.83

Table 4. Hypervolume of NSFWA and other algorithms.

	NSFWA		*S*-MOFWA		NSGA-II		SPEA2		RVEA	
Func.	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SCH	**1.33E+01**	**3.79E−05**	1.32E+01	7.62E−03	1.33E+01	1.21E−03	1.30E+01	7.52E−02	1.32E+01	3.09E−03
KUR	3.68E+01	4.03E−02	**3.71E+01**	9.02E−02	3.70E+01	**1.20E−02**	2.84E+01	2.92E−02	3.66E+01	1.31E−01
ZDT1	**6.61E−01**	**3.24E−05**	6.54E−01	1.03E−04	6.60E−01	2.77E−04	6.52E−01	2.23E−03	6.60E−01	4.84E−04
ZDT2	**3.28E−01**	**1.18E−04**	3.27E−01	4.00E−04	3.27E−01	2.24E−04	3.20E−01	2.62E−03	3.27E−01	4.09E−04
ZDT3	**1.04E+00**	**9.71E−06**	1.04E+00	3.20E−04	1.04E+00	1.44E−04	6.74E−01	3.58E−01	1.04E+00	3.76E−04
ZDT6	3.21E−01	7.93E−04	3.20E−01	5.32E−04	3.21E−01	3.26E−04	3.13E−01	3.43E−03	**3.22E−01**	**5.26E−05**
AR	1.50	1.83	2.50	3.16	2.67	2.00	5.00	4.50	3.33	3.50



(a) SCH (b) KUR (c) ZDT1

(d) ZDT2 (e) ZDT3 (f) ZDT6

Fig. 4. Solution set obtained by NSFWA.

5 Conclusion

In this paper, a novel multi-objective FWA named non-dominated sorting based fireworks algorithm is proposed. Non-dominated sorting based selection operator updates the external archive and selects fireworks according to the dominance relation and density to improve the diversity of solution set. Then, a multi-objective guided mutation operator is used to generate elite individuals for each populations to accelerate the convergence and enhance the stability. In order to further boost the performance of NSFWA on the problem that optimums locate near the bound, a novel mapping rule named midpoint mapping was proposed. Experiments on several test functions with different properties indicate that NSFWA has good performance on kinds of multi-optimization problems.

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