Fireworks Algorithm and Its Variants for Solving ICSI2014 Competition Problems

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Abstract. Firework algorithm (FWA) is a newly proposed swarm intelligence based optimization technique, which presents a different search manner by simulating the explosion of fireworks to search within the potential space till the terminal criterions are met. Since its introduction, a lot of improved work have been conducted, including the enhanced fireworks algorithm (EFWA), the dynamic search in FWA (dynFWA) and adaptive fireworks algorithm (AFWA). This paper is to use the FWA and its variants to take participate in the ICSI2014 competition, the performance among them are compared, and results on 2-, 10-, 30-dimensional benchmark functions are recorded.

Keywords: ICSI2014 competition, FWA, EFWA, dynFWA, AFWA.

1 Introduction

FWA is a population based swarm intelligence algorithm proposed by Tan and Zhu [16] in 2010. It takes the inspiration from the phenomenon that the fireworks explode and illuminate the local space around the fireworks in the night sky. Its proposed explosion search manner for each firework and cooperative strategy for allocating the resources among the fireworks swarm make it a novel and promising algorithm.

Assume that objective function f is a minimization problem with the form $\min_{x \in \Omega} f(x)$, and Ω is the feasible search region. The conventional FWA works as follows: At first, a fixed number of fireworks (N) are initialized within the feasible search range, and the quality of the fireworks' positions are evaluated, based on which the explosion amplitudes and explosion sparks number are calculated. Here, the principle idea for calculating them is that: the firework with smaller fitness will have larger number of explosion sparks and smaller explosion amplitude, while the firework with larger fitness will have smaller number of explosion sparks and bigger explosion amplitude. In addition, to increase the diversity of the population of the fireworks and explosion sparks, Gaussian mutation sparks are also introduced. After these operations of generating explosion and Gaussian mutation sparks, selection strategy is performed among the candidates set which includes fireworks, explosion sparks and Gaussian mutation

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sparks, and a fixed number of (N) fireworks are selected for the next iteration. The algorithm continues the search until the termination criterions are reached.

Since its first presentation in [16], FWA has attracted a number of researchers to develop the conventional algorithm and apply the algorithm for optimization of real world problems. For the algorithm developments, it includes the single objective algorithm developments [13] [12] [18] [14] [11], multi-objective algorithm developments [21], hybrid version with other algorithms [20] [2] [4] and parallel implementation versions [3]. For the application, FWA has been applied for FIR and IIR digital filters design [4], the initialization of Non-negative Matrix Factorization (NMF) and iterative optimization of NMF [10], [8], [9], spam detection [5], finger-vein identification [19] and power system reconfiguration [7] [6]. Experimental results suggest that FWA is a promising swarm intelligence algorithm, which needs further research and developments.

Motivation and Synopsis: The original motivation of this paper is to let FWA and its variants to participate the competition in ICSI2014 competition, and the performance among some typical improved work are compared. The remainder of this paper is organized as follows: Section 2 briefly introduces the framework of conventional fireworks algorithm, and the FWA variants are presented in Section 3, Experiments are given in Section 4 and finally conclusions are drawn in Section 5.

2 The Conventional FWA

In FWA, it works with a population of fireworks which can generate the explosion sparks and Gaussian mutation sparks thus to maintain the fireworks swarm with global and local search abilities. After generating two kinds of sparks, the selection strategy is performed for the selection of fireworks to the next iteration. Algorithm 1 gives the framework of conventional FWA.

In FWA, to make a contract among the fireworks and balance between the exploration and exploitation capacities, the fireworks are designed to take different explosion amplitudes and explosion sparks number. Assume that the fireworks number is N, then for each firework, the explosion sparks number s_i and explosion amplitude A_i are calculated as following:

$$A_i = \hat{A} \cdot \frac{f\left(X_i\right) - y_{min} + \varepsilon}{\sum_{i=1}^{N}\left(f\left(X_i\right) - y_{min}\right) + \varepsilon}, \tag{1}$$

$$s_i = M_e \cdot \frac{y_{max} - f(X_i) + \varepsilon}{\sum_{i=1}^{N}(y_{max} - f(X_i)) + \varepsilon}, \tag{2}$$

where, $y_{max} = max(f(X_i))$, $y_{min} = min(f(X_i))$, and $\hat{A}$ and M_e are two constants to control the explosion amplitude and the number of explosion sparks, respectively, and ε is the machine epsilon. In addition, to avoid the overwhelming effects of fireworks at good/bad locations, the max/min number of sparks are

Algorithm 1. General structure of conventional FWA

Initialize N fireworks X_i
repeat
  Explosion operator
    (i) Calculate explosion amplitude A_i and explosion sparks number s_i
    (ii) Generate the explosion sparks
  for each firework X_i, perform s_i times **do**
    Initialize the location of the "explosion sparks": $\hat{X}_i = X_i$
    Calculate offset displacement: $\triangle X = A_i \times rand(-1, 1)$
    $z = round(D * rand(0, 1))$
    Randomly select z dimensions of $\hat{X}_i$
    for each select dimension of $\hat{X}_i^k$ **do**
      $\hat{X}_i^k = \hat{X}_i^k + \triangle X$
      if $\hat{X}_i^k$ out of bounds **then**
        $\hat{X}_i^k = X_{min}^k + |\hat{X}_i^k| \ \% \ (X_{max}^k - X_{min}^k)$
      end if
    end for
      (iii) Evaluate fitness of newly created explosion sparks
  end for
  Gaussian mutation operator
    (i) Generate the Gaussian sparks
  for perform M_g times **do**
    Randomly initialize the location of the "Gaussian sparks": $\tilde{X}_i = X_i$
    Calculate offset displacement: $e = Gaussian(1, 1)$
    Set $z^k = round(rand(0, 1))$, $k = 1, 2, ..., d$
    for each dimension of $\hat{X}_i^k$, where $z^k == 1$ **do**
      $\tilde{X}_i^k = \tilde{X}_i^k \times e$
      if $\tilde{X}_i^k$ out of bounds **then**
        $\tilde{X}_i^k = X_{min}^k + |\tilde{X}_i^k| \ \% \ (X_{max}^k - X_{min}^k)$
      end if
    end for
  end for
    (ii) Evaluate fitness of newly created Gaussian sparks
  Selection strategy
    (i) Select fireworks for next iteration
until termination is met.

bounded by

$$s_i = \begin{cases} round(aM_e) & \text{if } s_i < aM_e, \\ round(bM_e) & \text{if } s_i > bM_e, \\ round(s_i) & otherwise. \end{cases} \tag{3}$$

where, a and b are constant parameters which confine minimal/maximal sparks number (the range of the sparks number). Then for each firework, the explosion sparks are generated according to Algorithm 1.

To increase the diversity, Gaussian mutation sparks are generated based on a Gaussian mutation operator (cf. Algorithm 1).

To retain the information to the next iteration, selection strategy is performed as most of the swarm intelligence algorithms and evolutionary algorithms. In the candidates set, the individual with minimal fitness is always selected while for the rest x_i in candidates set, the selection probability p_i is calculated as

$$p(x_i) = \frac{R(x_i)}{\sum_{j \in K} R(x_j)} \tag{4}$$

$$R(x_i) = \sum_{j \in K} d(x_i, x_j) = \sum_{j \in K} ||x_i - x_j|| \tag{5}$$

where K is the set of all current locations including original fireworks and both types of sparks.

3 The Selected Typical Improvement Work

3.1 Enhanced Fireworks Algorithm

Although FWA has shown its great performance when dealing with function optimization in [16], which outperforms SPSO [1] and CPSO [15] in the selected benchmark functions, in [18], Zheng et al presented a comprehensive study of operators in conventional FWA and proposed the enhanced FWA (EFWA). Some details of the EFWA are as following.

Amplitude of Explosion: In FWA, the explosion amplitude of the best firework is usually very close to 0. In EFWA, a lower bound A_{min} is introduced to avoid this problem:

$$A_i = max(A_i, A_{min}), \tag{6}$$

and A_{min} is non-linearly decreased with the evaluation times going on:

$$A^k_{\min}(t) = A_{init} - \frac{A_{init} - A_{final}}{evals_{max}} \sqrt{(2 * evals_{max} - t)t}, \tag{7}$$

where A_{init} and A_{final} are the initial and final minimum explosion amplitude, $evals_{max}$ is the maximum evaluation times and t is the current evaluation times.

Generating Sparks: In FWA, the number of to-be-mutated dimensions is calculated first and the displacement is calculated just once then used for all the selected dimensions. While in EFWA, for each dimension, an independent displacement $\varDelta X^k_i = A_i \times U(-1, 1)$ is calculated with the selection probability $U(0, 1)$, ($U(a, b)$ denotes the generated random number is under the mean distribution between a and b).

$$\hat{X^k_i} = X^k_i + \varDelta X^k_i. \tag{8}$$

Moreover, the way to generate Gaussian sparks makes use of the currently best location X_B to avoid the concentrated search on origin region.

$$\tilde{X}_i^k = X_i^k + e * (X_B^k - X_i^k), \tag{9}$$

where $e \sim N(0,1)$.

A new-generated spark will be mapped into a random place in the variable space with uniform distribution if the generated location exceeds the bounds.

$$\hat{X}_i^k = U(X_k^{min}, X_k^{max}). \tag{10}$$

Selection Strategy: To decrease the computational cost of selection strategy in FWA (Eq. 4), in EFWA, the best of the set will be selected first while the rest are randomly selected.

3.2 The dynFWA and AFWA

In FWA and EFWA, the explosion amplitude for fireworks is one of the most key features relevant to the performance. For each firework, its explosion amplitude is calculated by Eq. 1. In fact, the fitness of firework's position is only one kind of information to characterize the local information around X_i, good position needs further local search. However, for an optimization problem, the optimization process is dynamic, the previous static explosion amplitude calculation strategy only suggests that positions of fireworks are good or bad, not the local region within the positions of fireworks. So the explosion amplitude calculation method will lead to a bad local search ability and the experimental results in [18] are consistent with this idea.

For simplicity, the firework with minimal fitness in the fireworks swarm is defined as the core firework (CF, X_{CF}), which has the property that (*i*) its fitness is best among the fireworks, (*ii*) it is always selected to the next iteration. To overcome the limitations presented above, the dynamic search in FWA (dynFWA) and adaptive fireworks algorithm (AFWA) in [14] [11] are proposed respectively.

The dynFWA – Dynamic Explosion Amplitude Strategy: Assume that $f(\hat{X}_{best})$ is the minimal fitness among the explosion sparks and $f(X_{CF})$ denotes the fitness of CF. Here, in dynFWA, it concerns whether the generated explosion sparks can get better fitness than the CF, i.e. the $\Delta_f = f(\hat{X}_{best}) - f(X_{CF})$.

1) $\Delta_f < 0$

It means at least one of the newly generated sparks has smaller fitness than CF's fitness. If so, the $\hat{X}_{best}$ is probably created by the CF or the rest of fireworks other than CF. If $\hat{X}_{best}$ is created by CF, in order to speedup the search for the global optimum, the explosion amplitude of CF will become a bigger one compared with the current value. If $\hat{X}_{best}$ is created by one firework (X_i) other than CF, it has a high chance that the X_i is close to X_{CF}. If X_i is close to X_{CF}, the same explosion amplitude strategy for the CF in the next iteration is taken. If X_i is not close to X_{CF}, then the current explosion amplitude is in fact not effective for the newly generated CF for search any more. However, as it is hard to define the closeness and it is believed that the dynamic explosion amplitude strategy has its ability to adjust the explosion amplitude itself in the following iterations, so dynFWA just sets the explosion amplitude of newly selected CF with a increasing amplitude.

2) $\Delta_f \geq 0$
It means that none of the explosion sparks has found a position with better fitness compared to the CF. The reason for this situation is that the explosion amplitude of firework is too big for CF to search a better position. The CF needs to narrow down the search range. That is to reduce the explosion amplitude thus increasing the probability that the fireworks swarm can find a better position.

In fact, if the CF is far away from the global optimal position, increasing the explosion amplitude is one of the most effective methods to speedup the convergence. The reduction of the explosion amplitude makes it move towards the global optimal position, i.e. the CF finding a better solution.

In FWA and EFWA, to increase the diversity of the fireworks swarm, Gaussian mutation sparks are introduced. However, due to the selection method, the Gaussian mutation sparks do not work effectively as they are designed to, thus in dynFWA [14], they are eliminated.

The AFWA – Adaptive Fireworks Algorithm: The motivation of AFWA is to guarantee the progress made in current iteration is bigger than in the previous iteration [11].

AFWA tries to find the spark whose fitness is the minimal among the candidates whose fitness is worse than CF, and whose Infinite Norm is closest to the best candidate (i.e., Core firework or best spark), then the Infinite Norm between the found candidate with the firework will be taken as the explosion amplitude for the next iteration.

Under this explosion amplitude updating strategy, there are two cases. The first one is that the CF does not generate any good sparks whose fitness is smaller than the firework, then the fitness of all the sparks are larger than the firework, and the explosion amplitude will take the Infinite norm between the firework and the selected candidate. The explosion amplitude in the next iteration will be reduced. For the second case, it has two situations, and the explosion amplitude will be amplified according to the simulation with high chance. Moreover, as the Infinite Norm between the calculated candidate and the firework may change radically, in AFWA, it introduces the smoothing strategy.

4 Experiments

4.1 Experimental Setup

For the implementation of FWA, EFWA, dynFWA and AFWA in this paper, all the parameters are taken from [14] without any modifications. The experimental

Table 1. Run time results on f_9

	T1(s)	T2(s)	(T2-T1)/T1
dynFWA	28.0029	28.8269	0.0294
AFWA	28.0029	28.4186	0.0148

platform used in the experiments is MATLAB 2011b (Windows 7; Intel Core i7-2600 CPU @ 3.7 GHZ; 8 GB RAM) while ICSI-2014 competition problems are used as benchmark functions to compare the performance.

The description of the ICSI-2014 competition benchmark functions is as follows: It contains 30 functions, and for each function, the feasible range is set to $[-100, 100]$. Moreover, to make a comprehensive comparison, in the competition, three groups of experiments with dimension set as $D = 2, 10, 30$, and maximum evaluation times $D * 10000$ are designed. For each function, the max, min, mean, median value and standard deviation of 51 times results are recorded.

4.2 Experimental Results

The experimental results can be found in Table 2, Table 3 and Table 4. For the run time consuming, the experimental runs on f_9 of dynFWA and AFWA are given in Table 1 according to [17].

Table 2. Results for 2D functions

FWA	F	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Max	1.63E+03	1.89E+03	1.67E+00	1.13E+03	2.09E+00	1.68E-01	2.12E-03	2.58E+00	-3.74E+00	-4.14E-01	9.37E-03	6.62E+01	7.45E-02	8.05E-02	5.83E-02
	Min	8.03E-02	6.43E-01	1.67E+00	5.86E+00	1.39E-03	9.00E-05	2.00E-05	2.00E-05	-4.00E+00	-9.99E-01	1.90E-05	4.88E+01	1.90E-05	1.37E-03	3.40E-05
	Mean	1.97E+02	4.23E+02	1.67E+00	1.38E+02	3.39E-01	4.40E-02	6.91E-04	1.01E-01	-3.92E+00	-8.77E-01	1.04E-03	5.54E+01	2.10E-02	9.88E-03	1.03E-02
	Median	8.13E+01	2.10E+02	1.67E+00	7.99E+01	2.28E-01	2.37E-02	5.77E-04	4.16E-02	-3.93E+00	-9.10E-01	6.47E-04	5.48E+01	1.94E-02	4.46E-03	4.66E-03
	Std	2.96E+02	4.65E+02	2.26E-04	1.83E+02	3.94E-01	4.37E-02	4.89E-04	3.59E-01	6.26E-02	1.17E-01	1.44E-03	4.73E+00	1.14E-02	1.51E-02	1.30E-02
	F	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	Max	4.79E-02	5.94E-01	-6.01E+02	8.60E-03	0.00E+00	2.01E-01	1.62E-01	1.27E+01	-4.01E+00	-1.37E+00	1.03E+00	-3.62E+07	-4.82E+00	2.00E+01	1.01E+00
	Min	1.13E-03	2.32E-04	-8.38E+02	9.00E-06	0.00E+00	2.60E-05	2.00E-06	8.97E+00	-4.52E+00	-1.71E+00	5.74E-01	-3.74E+07	-5.86E+00	2.00E+01	2.68E-01
	Mean	1.54E-02	1.27E-01	-7.89E+02	2.71E-03	0.00E+00	4.35E-02	1.95E-02	9.36E+00	-4.31E+00	-1.50E+00	7.15E-01	-3.72E+07	-5.45E+00	2.00E+01	4.18E-01
	Median	1.34E-02	7.02E-02	-8.38E+02	2.81E-03	0.00E+00	7.93E-03	7.86E-03	8.98E+00	-4.31E+00	-1.39E+00	6.85E-01	-3.73E+07	-5.45E+00	2.00E+01	2.96E-01
	Std	1.10E-02	1.38E-01	9.06E+01	2.19E-03	0.00E+00	5.57E-02	3.19E-02	8.58E-01	1.16E-01	1.41E-01	9.67E-02	2.22E+05	2.54E-01	6.07E-03	2.17E-01
EFWA	F	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Max	7.31E+02	4.06E+03	1.67E+00	1.27E+03	7.12E-04	7.14E-01	2.28E-02	2.58E+00	-3.99E+00	-6.65E-01	3.13E-06	8.33E+01	3.56E-01	3.16E-01	4.78E-01
	Min	1.16E-04	1.27E-02	1.67E+00	7.69E-01	2.32E-06	2.47E-09	7.02E-07	3.44E-04	-4.00E+00	-1.00E+00	4.02E-10	4.86E+01	6.98E-08	2.30E-02	2.11E-03
	Mean	1.32E+02	1.01E+03	1.67E+00	3.43E+02	8.86E-05	1.40E-02	5.13E-03	5.37E-02	-3.99E+00	-9.45E-01	5.37E-07	5.14E+01	5.16E-02	1.43E-01	4.00E-02
	Median	2.73E+01	6.52E+02	1.67E+00	2.26E+02	5.91E-05	1.23E-06	3.69E-04	2.19E-03	-3.99E+00	-9.70E-01	2.50E-07	4.89E+01	1.94E-02	1.45E-01	3.02E-02
	Std	1.92E+02	1.13E+03	2.93E-09	3.33E+02	1.14E-04	1.00E-01	9.23E-03	3.61E-01	2.63E-03	7.60E-02	6.92E-07	8.41E+00	6.61E-02	7.72E-02	6.60E-02
	F	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	Max	1.88E-04	1.59E-05	-6.01E+02	4.86E-03	6.14E-32	1.08E-07	1.04E-05	1.07E+01	-2.11E+00	7.38E-01	1.31E+00	-1.46E+07	-4.51E+00	2.00E+01	1.12E+00
	Min	1.21E-05	2.68E-08	-8.38E+02	4.91E-10	1.59E-58	3.07E-11	4.61E-09	8.97E+00	-4.52E+00	-1.71E+00	5.72E-01	-3.74E+07	-5.74E+00	1.99E+01	2.67E-01
	Mean	8.94E-05	3.09E-06	-6.11E+02	5.68E-04	1.92E-33	1.46E-08	1.54E-06	9.01E+00	-4.09E+00	-1.32E+00	7.65E-01	-3.01E+07	-5.08E+00	2.00E+01	6.71E-01
	Median	8.51E-05	1.95E-06	-6.01E+02	1.16E-06	6.74E-39	4.82E-09	5.32E-07	8.97E+00	-4.34E+00	-1.39E+00	7.30E-01	-2.36E+07	-5.07E+00	2.00E+01	1.01E+00
	Std	4.64E-05	3.62E-06	4.64E+01	1.57E-03	9.39E-33	2.15E-08	2.04E-06	2.41E-01	6.85E-01	4.69E-01	1.69E-01	7.19E+06	2.68E-01	7.74E-03	3.76E-01
dynFWA	F	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Max	1.07E+03	5.40E+03	1.67E+00	1.45E+03	3.29E-02	1.81E-02	2.36E-02	1.60E-01	-3.87E+00	-9.13E-01	1.47E-03	5.33E+01	1.94E-02	2.85E-01	6.37E-02
	Min	3.37E-04	6.11E-02	1.67E+00	1.61E-02	5.16E-05	2.33E-07	2.39E-05	2.45E-04	-4.00E+00	-1.00E+00	1.42E-08	4.86E+01	1.90E-06	2.84E-03	7.42E-05
	Mean	1.56E+02	4.90E+02	1.67E+00	1.79E+02	3.85E-03	1.21E-03	9.70E-04	2.45E-02	-3.98E+00	-9.76E-01	7.02E-05	4.93E+01	1.55E-02	6.47E-02	1.74E-02
	Median	6.55E+01	8.70E+01	1.67E+00	3.54E+01	1.47E-03	1.21E-04	3.17E-04	1.05E-02	-3.99E+00	-9.80E-01	1.38E-05	4.91E+01	1.94E-02	4.13E-02	1.30E-02
	Std	2.44E+02	9.25E+02	1.16E-06	3.18E+02	6.32E-03	2.85E-03	3.30E-03	3.95E-02	2.45E-02	2.08E-02	2.09E-04	7.73E-01	7.75E-03	6.18E-02	1.55E-02
	F	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	Max	1.04E-02	7.81E-02	-8.38E+02	5.10E-03	0.00E+00	9.99E-02	1.59E-02	9.13E+00	-4.22E+00	-1.39E+00	9.18E-01	-3.73E+07	-4.73E+00	2.00E+01	1.01E+00
	Min	1.19E-04	7.38E-07	-8.38E+02	4.42E-07	0.00E+00	1.97E-08	1.07E-06	8.97E+00	-4.53E+00	-1.71E+00	5.76E-01	-3.74E+07	-5.89E+00	2.00E+01	2.67E-01
	Mean	2.31E-03	3.63E-03	-8.38E+02	1.14E-03	0.00E+00	1.98E-03	2.27E-03	8.99E+00	-4.42E+00	-1.57E+00	6.68E-01	-3.74E+07	-5.24E+00	2.00E+01	3.39E-01
	Median	1.83E-03	1.54E-04	-8.38E+02	4.24E-04	0.00E+00	4.57E-06	5.31E-04	8.98E+00	-4.46E+00	-1.71E+00	6.45E-01	-3.74E+07	-5.23E+00	2.00E+01	2.72E-01
	Std	2.16E-03	1.24E-02	3.38E-04	1.63E-03	0.00E+00	1.40E-02	4.05E-03	3.21E-02	9.62E-02	1.59E-01	7.80E-02	9.24E+02	2.50E-01	5.86E-03	1.45E-01
AFWA	F	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Max	1.97E+03	4.52E+03	1.67E+00	1.31E+03	3.04E-03	1.27E+01	2.38E-03	4.44E-15	-3.86E+00	-8.82E-01	2.74E-04	5.82E+01	1.94E-02	2.84E-01	7.07E-02
	Min	5.52E-02	5.52E-02	1.67E+00	5.15E-04	5.37E-07	1.42E-10	0.00E+00	8.88E-16	-4.00E+00	-1.00E+00	1.07E-14	4.86E+01	0.00E+00	5.11E-04	1.89E-05
	Mean	2.90E+02	1.03E+03	1.67E+00	3.00E+02	4.86E-04	5.61E-01	1.35E-04	1.10E-15	-3.99E+00	-9.71E-01	1.98E-05	5.04E+01	1.83E-02	5.75E-02	2.21E-02
	Median	1.32E+02	6.46E+02	1.67E+00	1.66E+02	3.05E-04	1.93E-05	9.65E-18	8.88E-16	-4.00E+00	-9.80E-01	8.55E-07	4.89E+01	1.94E-02	3.36E-02	1.51E-02
	Std	4.12E+02	1.23E+03	4.50E-09	3.53E+02	5.87E-04	2.07E+00	4.24E-04	8.44E-16	2.08E-02	2.56E-02	4.87E-05	2.76E+00	4.62E-03	7.09E-02	2.05E-02
	F	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	Max	0.00E+00	0.00E+00	-6.01E+02	4.85E-03	0.00E+00	0.00E+00	0.00E+00	1.07E+01	-3.12E+00	-6.50E-01	1.31E+00	-3.74E+07	-5.04E+00	2.00E+01	5.57E-01
	Min	0.00E+00	0.00E+00	-8.38E+02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	8.97E+00	-4.53E+00	-1.71E+00	5.71E-01	-3.74E+07	-5.93E+00	2.00E+01	2.67E-01
	Mean	0.00E+00	0.00E+00	-8.29E+02	1.22E-03	0.00E+00	0.00E+00	0.00E+00	9.01E+00	-4.35E+00	-1.49E+00	7.05E-01	-3.74E+07	-5.53E+00	2.00E+01	3.11E-01
	Median	0.00E+00	0.00E+00	-8.38E+02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	8.97E+00	-4.43E+00	-1.39E+00	6.67E-01	-3.74E+07	-5.54E+00	2.00E+01	2.70E-01
	Std	0.00E+00	0.00E+00	4.45E+01	2.03E-03	0.00E+00	0.00E+00	0.00E+00	2.41E-01	2.61E-01	1.94E-01	1.36E-01	8.99E+01	2.46E-01	8.51E-03	7.74E-02

Table 3. Results for 10D functions

	F	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
FWA	Max	7.42E+06	1.35E+05	1.82E+02	3.52E+02	4.59E+00	1.63E+02	6.03E-02	7.97E+00	-1.51E+01	-3.20E+00	2.57E+01	6.15E-01	1.76E+00	2.06E-01	2.01E-01
	Min	2.25E+05	2.49E+03	1.71E+02	2.28E+01	9.06E-02	5.90E+00	9.08E-03	2.86E+00	-1.88E+01	-8.05E+00	2.36E-01	4.35E-01	2.19E-01	1.28E-02	1.12E-02
	Mean	2.78E+06	4.15E+04	1.74E+02	1.19E+02	1.95E+00	3.83E+01	3.39E-02	5.01E+00	-1.70E+01	-5.67E+00	8.25E+00	5.35E-01	8.91E-01	9.38E-02	7.14E-02
	Median	2.32E+06	3.35E+04	1.74E+02	1.01E+02	1.57E+00	2.84E+01	3.36E-02	4.83E+00	-1.72E+01	-5.77E+00	7.47E+00	5.36E-01	8.52E-01	8.77E-02	5.92E-02
	Std	1.81E+06	2.95E+04	2.57E+00	6.01E+01	1.23E+00	3.35E+01	1.47E-02	1.30E+00	9.77E-01	1.04E+00	6.28E+00	3.99E-02	3.02E-01	4.60E-02	4.34E-02
	F	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	Max	1.05E+00	2.54E+01	-1.66E+03	9.41E-01	4.65E+00	2.20E+00	1.71E+01	3.22E+01	-2.10E+01	4.54E+01	1.53E-02	-1.47E+07	-5.16E+00	2.00E+01	1.08E+00
	Min	9.41E-02	1.65E+00	-4.12E+03	3.36E-02	1.92E-05	9.99E-02	2.61E-01	2.09E+01	-3.27E+01	4.31E+01	8.55E-03	-9.88E+07	-5.93E+00	2.00E+01	1.01E+00
	Mean	4.22E-01	1.14E+01	-2.92E+03	2.65E-01	5.86E-01	7.65E-01	4.39E+00	2.63E+01	-2.78E+01	4.36E+01	1.12E-02	-5.30E+07	-5.53E+00	2.00E+01	1.03E+00
	Median	3.76E-01	1.16E+01	-2.95E+03	1.95E-01	5.78E-02	7.00E-01	3.30E+00	2.70E+01	-2.81E+01	4.35E+01	1.12E-02	-4.67E+07	-5.53E+00	2.00E+01	1.04E+00
	Std	2.07E-01	6.18E+00	1.04E+03	2.09E-01	1.02E+00	4.65E-01	3.29E+00	2.58E+00	2.66E+00	4.20E-01	1.77E-03	2.44E+07	1.97E-01	1.74E-03	1.50E-02
EFWA	F	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Max	1.37E+05	2.00E+04	1.70E+02	6.60E+01	2.53E-01	1.00E+01	3.07E+00	2.01E+01	-1.22E+01	8.07E+00	4.25E+01	5.84E-01	3.26E+00	2.02E+00	1.67E+00
	Min	1.73E+04	7.47E+02	1.70E+02	3.62E+00	1.07E-02	2.94E+00	1.98E-03	2.99E+00	-1.57E+01	-7.25E+00	3.65E+00	3.90E-01	6.40E-01	8.65E-02	8.09E-02
	Mean	5.83E+04	7.52E+03	1.70E+02	1.89E+01	1.12E-01	7.81E+00	8.51E-01	1.66E+01	-1.48E+01	-2.16E+00	2.04E+01	4.96E-01	2.11E+00	7.24E-01	3.02E-01
	Median	5.47E+04	6.49E+03	1.70E+02	1.71E+01	1.24E-01	8.46E+00	7.29E-01	2.00E+01	-1.51E+01	-2.53E+00	1.79E+01	5.03E-01	2.12E+00	3.40E-01	3.45E-01
	Std	2.81E+04	4.46E+03	4.11E-02	1.37E+01	6.73E-02	1.74E+00	9.63E-01	5.97E+00	8.49E-01	3.63E+00	1.05E+01	4.51E-02	5.64E-01	7.60E-01	2.40E-01
	F	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	Max	1.46E+00	5.94E-02	-3.00E+03	4.82E+00	4.84E+02	6.00E-01	2.28E-01	5.51E+01	-8.86E+00	5.29E+01	1.37E-02	-3.68E+07	-5.13E+00	2.00E+01	1.08E+00
	Min	3.79E-02	7.01E-03	-3.00E+03	1.51E-02	0.00E+00	9.99E-02	2.64E-03	2.36E+01	-3.26E+01	4.29E+01	7.05E-03	-1.07E+08	-5.66E+00	2.00E+01	1.01E+00
	Mean	2.45E-01	2.44E-02	-3.00E+03	8.23E-01	9.57E+00	2.29E-01	9.15E-02	3.88E+01	-2.18E+01	4.37E+01	9.84E-03	-9.75E+07	-5.37E+00	2.00E+01	1.04E+00
	Median	2.11E-01	2.13E-02	-3.00E+03	4.68E-01	3.90E-03	2.00E-01	9.34E-02	3.95E+01	-2.21E+01	4.29E+01	9.81E-03	-1.06E+08	-5.36E+00	2.00E+01	1.04E+00
	Std	2.31E-01	1.26E-02	3.33E-02	9.42E-01	6.77E+01	1.25E-01	6.71E-02	7.04E+00	5.33E+00	2.62E+00	1.57E-03	1.87E+07	1.29E-01	4.34E-04	1.68E-02
dynFWA	F	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Max	1.32E+05	5.20E+04	1.70E+02	1.86E+02	2.00E-01	9.71E+00	3.32E-01	4.56E+00	-1.69E+01	-5.32E+00	1.52E+01	5.67E-01	1.66E+00	2.03E-01	1.89E-01
	Min	1.80E+03	6.97E+02	1.70E+02	4.78E+00	4.23E-03	8.88E-01	5.74E-04	2.19E-02	-1.94E+01	-8.73E+00	7.35E-04	3.89E-01	2.08E-01	2.83E-02	7.27E-03
	Mean	3.18E+04	9.49E+03	1.70E+02	6.48E+01	6.31E-02	6.56E+00	1.64E-02	2.03E+00	-1.84E+01	-7.24E+00	6.54E+00	4.70E-01	9.80E-01	8.83E-02	1.03E-01
	Median	2.49E+04	6.61E+03	1.70E+02	5.74E+01	5.46E-02	7.04E+00	4.19E-03	2.04E+00	-1.85E+01	-7.34E+00	5.86E+00	4.63E-01	9.96E-01	8.04E-02	1.04E-01
	Std	2.90E+04	9.87E+03	7.52E-02	4.78E+01	5.23E-02	2.51E+00	4.85E-02	1.23E+00	6.30E-01	7.17E-01	4.17E+00	4.42E-02	3.85E-01	4.11E-02	4.01E-02
	F	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	Max	8.67E-01	1.68E-02	-1.68E+03	1.66E-01	8.76E+00	8.00E-01	2.33E-01	4.20E+01	-2.70E+01	5.33E+01	9.77E-03	-1.91E+07	-5.23E+00	2.00E+01	1.04E+00
	Min	3.89E-02	7.06E-05	-4.14E+03	1.32E-03	6.79E-12	9.99E-02	5.62E-05	1.96E+01	-3.51E+01	4.29E+01	6.68E-03	-1.09E+08	-5.94E+00	2.00E+01	1.01E+00
	Mean	2.67E-01	2.57E-03	-2.31E+03	2.42E-02	3.95E-01	2.88E-01	4.80E-02	2.90E+01	-3.29E+01	4.36E+01	7.92E-03	-7.83E+07	-5.77E+00	2.00E+01	1.02E+00
	Median	2.21E-01	2.00E-03	-2.02E+03	1.50E-02	1.75E-03	3.00E-01	1.78E-02	2.87E+01	-3.33E+01	4.29E+01	7.90E-03	-6.71E+07	-5.81E+00	2.00E+01	1.04E+00
	Std	1.89E-01	2.70E-03	7.54E+02	3.26E-02	1.44E+00	1.48E-01	5.98E-02	5.18E+00	1.70E+00	2.41E+00	6.37E-04	2.89E+07	1.37E-01	1.28E-04	1.39E-02
AFWA	F	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Max	1.06E+06	4.49E+04	1.71E+02	3.50E+02	1.20E+00	8.13E+01	9.26E-01	7.06E+00	-1.57E+01	-3.80E+00	2.51E+01	5.96E-01	2.03E+00	2.11E-01	2.32E-01
	Min	2.07E+03	8.97E+02	1.70E+02	1.04E+01	1.12E-02	1.93E+00	1.69E-03	1.18E+00	-1.95E+01	-8.54E+00	1.85E-03	3.89E-01	3.24E-01	1.16E-02	1.48E-02
	Mean	3.05E+05	1.42E+04	1.70E+02	9.24E+01	2.10E-01	1.11E+01	6.83E-02	3.60E+00	-1.77E+01	-7.00E+00	1.17E+01	5.10E-01	1.27E+00	1.01E-01	1.21E-01
	Median	2.26E+05	1.03E+04	1.70E+02	6.25E+01	8.45E-02	8.53E+00	1.52E-02	3.22E+00	-1.77E+01	-7.17E+00	1.20E+01	5.11E-01	1.29E+00	9.84E-02	1.15E-01
	Std	2.84E+05	1.16E+04	3.96E-01	7.98E+01	2.59E-01	1.24E+01	1.87E-01	1.47E+00	8.05E-01	9.47E-01	6.82E+00	4.17E-02	3.85E-01	4.52E-02	5.26E-02
	F	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	Max	1.23E+00	1.66E+00	-1.23E+03	2.82E-01	2.09E+01	1.90E+00	1.27E+00	4.19E+01	-2.52E+01	5.34E+01	1.15E-02	-1.73E+07	-4.86E+00	2.00E+01	1.04E+00
	Min	5.55E-02	7.31E-02	-4.14E+03	1.69E-03	0.00E+00	9.99E-02	0.00E+00	2.00E+01	-3.50E+01	4.29E+01	6.74E-03	-1.06E+08	-5.92E+00	2.00E+01	1.01E+00
	Mean	3.41E-01	4.24E-01	-2.22E+03	5.90E-02	7.22E-01	5.19E-01	2.78E-01	3.16E+01	-3.10E+01	4.39E+01	8.69E-03	-5.37E+07	-5.70E+00	2.00E+01	1.03E+00
	Median	2.95E-01	3.67E-01	-2.02E+03	3.04E-02	5.00E-03	5.00E-01	1.66E-01	3.17E+01	-3.13E+01	4.29E+01	8.52E-03	-4.54E+07	-5.78E+00	2.00E+01	1.04E+00
	Std	2.47E-01	3.28E-01	7.49E+02	6.69E-02	3.20E+00	3.38E-01	3.17E-01	5.21E+00	2.02E+00	2.99E+00	9.22E-04	2.86E+07	2.15E-01	2.38E-04	1.33E-02

Table 4. Results for 30D functions

	F	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
FWA	Max	1.14E+07	5.51E+05	5.68E+03	2.25E+02	4.87E+00	2.06E+02	1.28E-01	7.67E+00	-4.62E+01	-1.26E+01	6.78E+00	1.54E-02	6.48E+00	7.64E-02	1.20E-01
	Min	2.79E+06	6.02E+04	4.79E+03	7.04E+01	1.07E+00	7.51E+01	3.25E-02	4.36E+00	-5.23E+01	-2.21E+01	1.59E+00	1.39E-02	1.48E+00	4.96E-03	8.57E-03
	Mean	6.27E+06	2.38E+05	5.16E+03	1.57E+02	2.61E+00	1.36E+02	6.65E-02	6.12E+00	-4.85E+01	-1.80E+01	4.38E+00	1.48E-02	3.68E+00	2.98E-02	3.30E-02
	Median	5.37E+06	1.98E+05	5.13E+03	1.55E+02	2.64E+00	1.31E+02	6.47E-02	6.19E+00	-4.86E+01	-1.81E+01	4.42E+00	1.48E-02	3.71E+00	2.85E-02	2.68E-02
	Std	2.36E+06	1.11E+05	2.24E+02	3.50E+01	8.88E-01	3.41E+01	2.08E-02	7.97E-01	1.37E+00	2.51E+00	1.37E+00	3.57E-04	1.19E+00	1.73E-02	2.32E-02
	F	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	Max	2.59E+00	1.07E+02	-8.94E+03	1.34E+00	3.87E+02	3.60E+00	1.91E+01	7.52E+01	-8.74E+01	1.40E+03	9.67E-04	-3.48E+08	-5.29E+00	2.00E+01	1.18E+00
	Min	8.97E-01	1.84E+01	-1.25E+04	3.11E-01	1.29E+00	1.10E+00	4.05E+00	3.67E+01	-1.02E+02	1.18E+03	4.77E-04	-7.54E+08	-5.94E+00	2.00E+01	1.08E+00
	Mean	1.68E+00	5.81E+01	-9.31E+03	7.27E-01	4.80E+01	2.08E+00	1.09E+01	5.72E+01	-9.50E+01	1.28E+03	6.19E-04	-5.17E+08	-5.61E+00	2.00E+01	1.10E+00
	Median	1.71E+00	5.63E+01	-8.97E+03	7.16E-01	3.36E+01	2.10E+00	1.09E+01	5.59E+01	-9.48E+01	1.28E+03	6.05E-04	-4.37E+08	-5.61E+00	2.00E+01	1.08E+00
	Std	3.89E-01	1.73E+01	1.04E+03	2.59E-01	6.31E+01	5.53E-01	3.37E+00	9.05E+00	3.35E+00	4.77E+01	8.01E-05	1.38E+08	1.63E-01	2.22E-04	2.66E-02
EFWA	F	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Max	3.62E+05	2.71E+04	4.53E+03	2.26E+01	1.62E-01	9.03E+01	3.79E+00	2.02E+01	-4.87E+00	-3.50E-01	1.53E+02	1.51E-02	1.20E+01	2.01E+00	1.92E+00
	Min	4.11E+03	3.27E+03	4.51E+03	2.97E+00	2.26E-02	2.70E+01	9.18E-03	6.94E+00	-4.75E+01	-1.98E+01	8.66E+00	1.29E-02	1.86E+00	5.92E-02	4.26E-02
	Mean	6.63E+04	1.23E+04	4.52E+03	8.91E+00	7.12E-02	3.03E+01	2.12E+00	1.92E+01	-1.07E+01	-1.27E+01	7.82E+01	1.41E-02	7.32E+00	1.94E+00	8.91E-01
	Median	4.60E+04	1.16E+04	4.52E+03	8.13E+00	6.62E-02	2.91E+01	1.99E+00	2.01E+01	-6.34E+00	-1.40E+01	7.43E+01	1.41E-02	7.52E+00	1.98E+00	4.62E-01
	Std	7.26E+04	5.59E+03	5.27E+00	4.43E+00	2.96E-02	8.60E+00	8.40E-01	2.93E+00	1.23E+01	4.22E+00	4.07E+01	4.78E-04	2.70E+00	2.69E-01	6.66E-01
	F	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	Max	1.35E+00	3.12E+00	-9.01E+03	4.99E+00	4.80E-05	1.20E+00	2.84E-01	1.26E+02	-7.58E+01	1.13E+03	4.73E-04	-5.08E+08	-5.31E+00	2.00E+01	1.18E+00
	Min	1.47E-01	1.39E-01	-9.01E+03	1.45E-01	3.00E-06	3.00E-01	1.63E-02	4.95E+01	-9.42E+01	1.12E+03	4.03E-04	-8.61E+08	-5.86E+00	2.00E+01	1.04E+00
	Mean	4.92E-01	1.07E+00	-9.01E+03	1.27E+00	1.20E-05	6.86E-01	6.72E-02	9.27E+01	-8.55E+01	1.13E+03	4.31E-04	-6.11E+08	-5.52E+00	2.00E+01	1.09E+00
	Median	4.11E-01	8.72E-01	-9.01E+03	7.90E-01	8.00E-06	7.00E-01	4.27E-02	9.27E+01	-8.60E+01	1.13E+03	4.28E-04	-5.21E+08	-5.50E+00	2.00E+01	1.08E+00
	Std	2.65E-01	6.63E-01	1.80E-01	1.26E+00	9.00E-06	2.10E-01	6.24E-02	2.04E+01	4.91E+00	1.30E+00	1.70E-05	1.50E+08	1.24E-01	2.31E-04	3.09E-02
dynFWA	F	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Max	2.43E+05	3.07E+04	4.55E+03	5.21E+01	2.43E-01	9.79E+01	3.16E-02	5.32E+00	-4.86E+01	-1.62E+01	5.21E+01	1.51E-02	5.04E+00	7.61E-02	4.73E-01
	Min	2.61E+03	4.01E+03	4.51E+03	1.04E+01	8.11E-03	2.74E+01	4.82E-03	9.59E-01	-5.63E+01	-2.61E+01	6.59E-04	1.31E-02	1.52E+00	2.39E-02	3.06E-02
	Mean	5.48E+04	1.26E+04	4.52E+03	3.03E+01	7.79E-02	3.64E+01	1.20E-02	2.75E+00	-5.33E+01	-2.22E+01	1.07E+01	1.41E-02	2.98E+00	4.61E-02	2.41E-01
	Median	3.41E+04	1.11E+04	4.52E+03	2.85E+01	6.34E-02	2.94E+01	1.15E-02	2.67E+00	-5.35E+01	-2.25E+01	2.50E+00	1.42E-02	3.00E+00	4.70E-02	1.11E-01
	Std	5.21E+04	6.72E+03	6.35E+00	1.14E+01	5.14E-02	2.01E+01	4.62E-03	8.24E-01	1.76E+00	2.02E+00	1.45E+01	5.07E-04	7.50E-01	1.31E-02	1.88E-01
	F	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	Max	1.21E+00	5.49E+00	-3.68E+03	9.42E-01	4.44E+01	2.00E+00	3.70E-01	7.09E+01	-9.33E+01	1.14E+03	4.72E-04	-3.95E+07	-5.38E+00	2.00E+01	1.13E+00
	Min	1.39E-01	1.73E-01	-6.06E+03	8.69E-03	2.36E-03	4.00E-01	2.52E-03	3.35E+01	-1.11E+02	1.12E+03	4.01E-04	-5.18E+08	-5.96E+00	2.00E+01	1.04E+00
	Mean	6.31E-01	2.17E+00	-5.03E+03	1.95E-01	2.76E+00	8.87E-01	7.58E-02	4.98E+01	-1.04E+02	1.13E+03	4.23E-04	-2.64E+08	-5.80E+00	2.00E+01	1.06E+00
	Median	5.94E-01	1.98E+00	-6.05E+03	1.29E-01	5.70E-01	9.00E-01	3.94E-02	4.97E+01	-1.05E+02	1.13E+03	4.19E-04	-2.90E+08	-5.84E+00	2.00E+01	1.04E+00
	Std	2.65E-01	1.17E+00	1.18E+03	2.15E-01	7.05E+00	3.07E-01	8.57E-02	9.16E+00	3.57E+00	3.47E+00	1.66E-05	1.02E+08	1.40E-01	2.14E-04	2.46E-02
AFWA	F	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Max	2.85E+05	2.81E+04	4.55E+03	7.17E+01	2.00E-01	9.34E+01	4.94E-02	5.32E+00	-4.81E+01	-1.47E+01	4.20E+01	1.50E-02	5.82E+00	6.27E-02	4.80E-01
	Min	7.49E+02	2.83E+03	4.51E+03	1.43E+01	1.42E-02	2.69E+01	6.80E-03	2.02E+00	-5.62E+01	-2.59E+01	6.37E-03	1.31E-02	1.34E+00	1.80E-02	1.95E-02
	Mean	5.14E+04	1.19E+04	4.52E+03	3.01E+01	7.07E-02	3.26E+01	1.50E-02	3.19E+00	-5.26E+01	-2.18E+01	5.94E+00	1.42E-02	3.11E+00	4.00E-02	3.07E-01
	Median	1.97E+04	1.02E+04	4.52E+03	2.69E+01	5.79E-02	2.92E+01	1.40E-02	3.03E+00	-5.28E+01	-2.20E+01	1.35E-01	1.42E-02	2.97E+00	3.95E-02	4.22E-01
	Std	6.60E+04	5.84E+03	7.28E+00	1.18E+01	4.48E-02	1.48E+01	6.38E-03	7.93E-01	1.87E+00	2.03E+00	8.29E+00	3.87E-04	9.45E-01	1.20E-02	1.87E-01
	F	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	Max	1.44E+00	6.34E+00	-3.69E+03	6.64E-01	5.80E+01	1.80E+00	1.87E-01	7.59E+01	-9.54E+01	1.14E+03	4.51E-04	-7.18E+07	-5.37E+00	2.00E+01	1.13E+00
	Min	2.31E-01	5.86E-01	-9.01E+03	1.06E-02	7.86E-04	3.00E-01	3.93E-03	3.32E+01	-1.10E+02	1.12E+03	4.03E-04	-8.56E+08	-5.98E+00	2.00E+01	1.04E+00
	Mean	7.35E-01	2.40E+00	-5.39E+03	1.33E-01	3.70E+00	9.78E-01	4.91E-02	5.11E+01	-1.04E+02	1.13E+03	4.23E-04	-3.07E+08	-5.81E+00	2.00E+01	1.07E+00
	Median	7.15E-01	2.21E+00	-6.05E+03	8.82E-02	3.90E-01	1.00E+00	3.38E-02	5.02E+01	-1.04E+02	1.13E+03	4.21E-04	-2.91E+08	-5.85E+00	2.00E+01	1.08E+00
	Std	2.83E-01	1.34E+00	1.45E+03	1.55E-01	9.30E+00	3.48E-01	4.71E-02	8.84E+00	3.47E+00	3.17E+00	1.20E-05	1.53E+08	1.36E-01	2.07E-04	2.76E-02

From the results in different dimension, it can be seen that with the increasing of the dimension, the results optimized by all the algorithms get worsen, which is usually called "*dimension of curse*". From the run time results in Table 1, it can be seen that AFWA achieve smaller $(T2 - T1)/T1$ than dynFWA. Here we also need to point out that the implementation of the code is one of the core factors to influence the run time.

From the results of 2D functions in Table 2, it can be seen that AFWA achieves better results than FWA, EFWA and dynFWA. Especially on f_{16}, f_{17}, f_{20}, f_{21}, f_{22}, AFWA gets the optimum of these functions. Table 3 gives the results of 10D functions. The dynFWA and AFWA still outperform EFWA and FWA. For the comparison between dynFWA and AFWA, dynFWA achieves smaller mean fitness. Table 4 shows the results on 30D functions. None of the algorithms works well, since all the maximum and minimum are different for each function. The dynFWA and AFWA still outperform EFWA and FWA due to their great local search ability, while the performances of dynFWA and AFWA do not differ much.

5 Conclusion

In this paper, the FWA and its variants are used to take the ICSI2014 competition for solving competition problems which contains 30 functions, and the three groups of experimental results with the dimensions set to $2, 10, 30$ are recorded. In the competition, the error smaller than $2^{-52} \approx 2.22e^{-16}$ is set to 0. It can be seen that for some functions, the most recent work dynFWA and AFWA still can not get the optimum, thus further research needs to be taken and it is believed that there is a long way to go for fireworks algorithm in the future.

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References

1. Bratton, D., Kennedy, J.: Defining a standard for particle swarm optimization. In: Swarm Intelligence Symposium, SIS 2007, pp. 120–127. IEEE (2007)
2. Yu, C., Kelley, L., Zheng, S.: Fireworks algorithm with differential mutation for solving the cec 2014 competition problems. In: 2014 IEEE Congress on Evolutionary Computation (CEC). IEEE (2014)
3. Ding, K., Zheng, S., Tan, Y.: A gpu-based parallel fireworks algorithm for optimization. In: Proceeding of the Fifteenth Annual Conference on Genetic and Evolutionary Computation Conference, GECCO 2013, pp. 9–16. ACM, New York (2013), `http://doi.acm.org/10.1145/2463372.2463377`
4. Gao, H., Diao, M.: Cultural firework algorithm and its application for digital filters design. International Journal of Modelling, Identification and Control 14(4), 324–331 (2011)

5. He, W., Mi, G., Tan, Y.: Parameter optimization of local-concentration model for spam detection by using fireworks algorithm. In: Tan, Y., Shi, Y., Mo, H. (eds.) ICSI 2013, Part I. LNCS, vol. 7928, pp. 439–450. Springer, Heidelberg (2013)
6. Imran, A.M., Kowsalya, M.: A new power system reconfiguration scheme for power loss minimization and voltage profile enhancement using fireworks algorithm. International Journal of Electrical Power & Energy Systems 62, 312–322 (2014)
7. Imran, A.M., Kowsalya, M., Kothari, D.: A novel integration technique for optimal network reconfiguration and distributed generation placement in power distribution networks. International Journal of Electrical Power & Energy Systems 63, 461–472 (2014)
8. Janecek, A., Tan, Y.: Iterative improvement of the multiplicative update nmf algorithm using nature-inspired optimization. In: 2011 Seventh International Conference on, Natural Computation (ICNC), vol. 3, pp. 1668–1672. IEEE (2011)
9. Janecek, A., Tan, Y.: Swarm intelligence for non-negative matrix factorization. International Journal of Swarm Intelligence Research (IJSIR) 2(4), 12–34 (2011)
10. Janecek, A., Tan, Y.: Using population based algorithms for initializing nonnegative matrix factorization. In: Tan, Y., Shi, Y., Chai, Y., Wang, G. (eds.) ICSI 2011, Part II. LNCS, vol. 6729, pp. 307–316. Springer, Heidelberg (2011)
11. Junzhi Li, S.Z., Tan, Y.: Adaptive fireworks algorithm. In: 2014 IEEE Congress on Evolutionary Computation (CEC). IEEE (2014)
12. Liu, J., Zheng, S., Tan, Y.: The improvement on controlling exploration and exploitation of firework algorithm. In: Tan, Y., Shi, Y., Mo, H. (eds.) ICSI 2013, Part I. LNCS, vol. 7928, pp. 11–23. Springer, Heidelberg (2013)
13. Pei, Y., Zheng, S., Tan, Y., Hideyuki, T.: An empirical study on influence of approximation approaches on enhancing fireworks algorithm. In: Proceedings of the 2012 IEEE Congress on System, Man and Cybernetics, pp. 1322–1327. IEEE (2012)
14. Zheng, S., Andreas, J., Li, J., Tan, Y.: Dynamic search in fireworks algorithm. In: 2014 IEEE Congress on Evolutionary Computation (CEC). IEEE (2014)
15. Tan, Y., Xiao, Z.: Clonal particle swarm optimization and its applications. In: IEEE Congress on Evolutionary Computation, CEC 2007, pp. 2303–2309. IEEE (2007)
16. Tan, Y., Zhu, Y.: Fireworks algorithm for optimization. In: Tan, Y., Shi, Y., Tan, K.C. (eds.) ICSI 2010, Part I. LNCS, vol. 6145, pp. 355–364. Springer, Heidelberg (2010)
17. Tan, Y., Li, J., Zheng, Z.: Icsi 2014 competition on single objective optimization (2014)
18. Zheng, S., Andreas, J., Tan, Y.: Enhanced fireworks algorithm. In: 2013 IEEE Congress on Evolutionary Computation (CEC), pp. 2069–2077. IEEE (2013)
19. Zheng, S., Tan, Y.: A unified distance measure scheme for orientation coding in identification. In: 2013 IEEE Congress on Information Science and Technology, pp. 979–985. IEEE (2013)
20. Zheng, Y., Xu, X., Ling, H.: A hybrid fireworks optimization method with differential evolution. Neurocomputing (2012)
21. Zheng, Y.J., Song, Q., Chen, S.Y.: Multiobjective fireworks optimization for variable-rate fertilization in oil crop production. Applied Soft Computing 13(11), 4253–4263 (2013)