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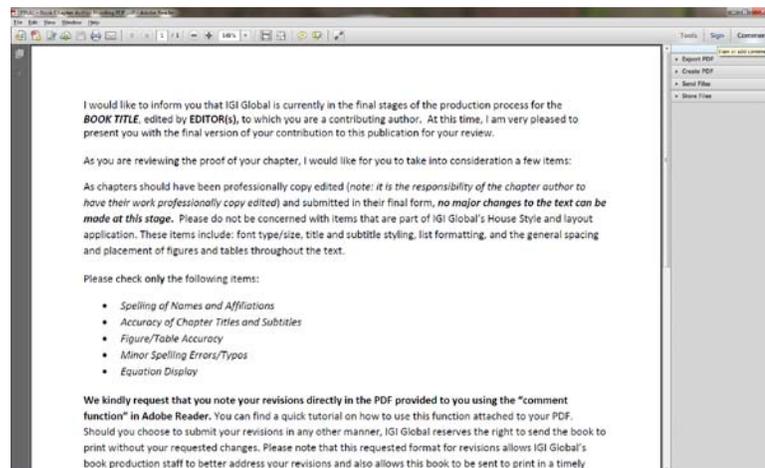
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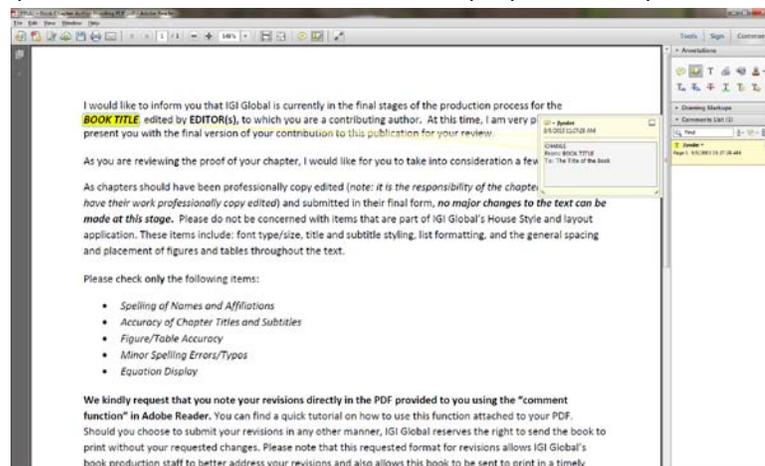
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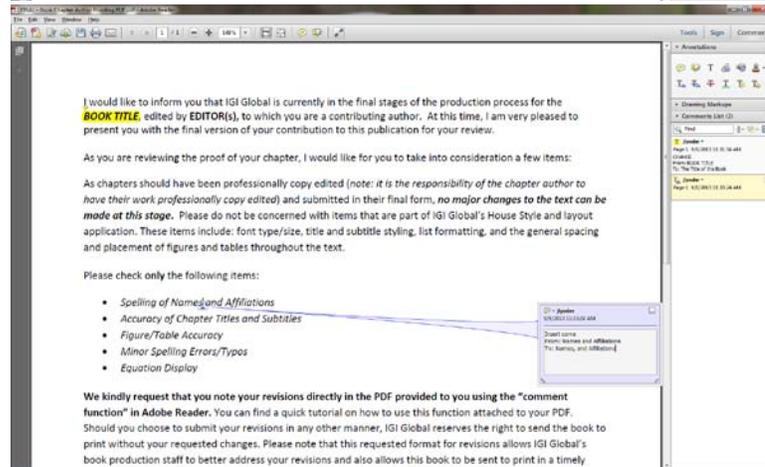
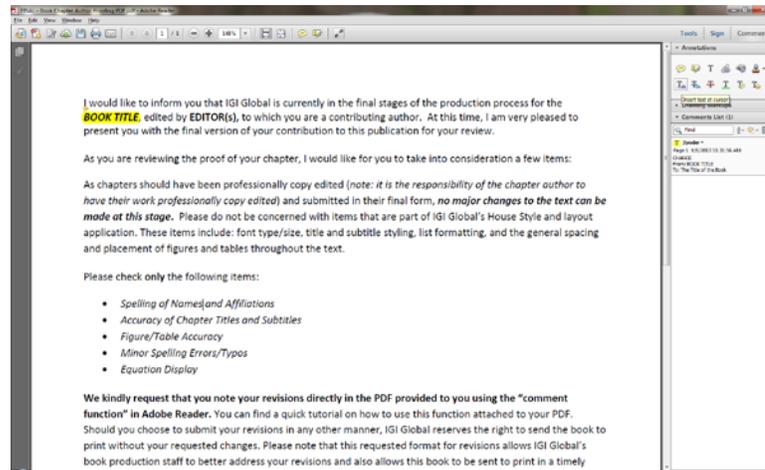
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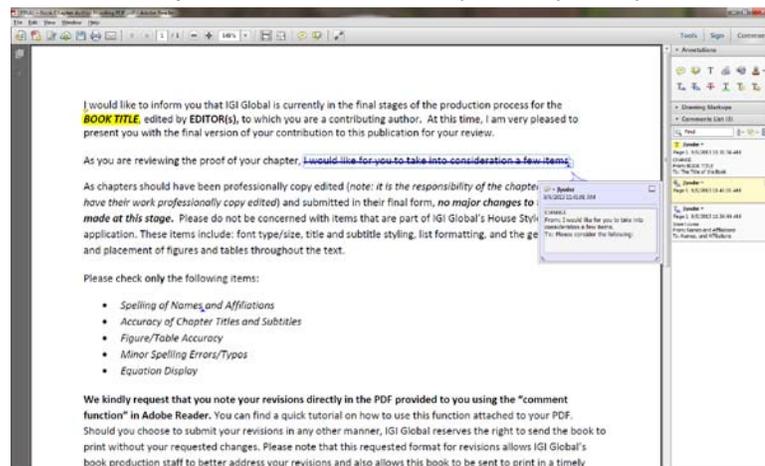
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IGI PUBLISHING
WWW.IGI-GLOBAL.COM

International Journal of Swarm Intelligence Research

October-December 2013, Vol. 4, No. 4

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Introduction to Fireworks Algorithm

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ABSTRACT

Inspired by fireworks explosion at night, conventional fireworks algorithm (FWA) was developed in 2010. Since then, several improvements and some applications were proposed to improve the efficiency of FWA. In this paper, the conventional fireworks algorithm is first summarized and reviewed and then three improved fireworks algorithms are provided. By changing the ways of calculating numbers and amplitudes of sparks in fireworks' explosion, the improved FWA algorithms become more reasonable and explainable. In addition, the multi-objective fireworks algorithm and the graphic processing unit (GPU) based fireworks algorithm are also presented, particularly the GPU based fireworks algorithm is able to speed up the optimization process considerably. Extensive experiments on 13 benchmark functions demonstrate that the three improved fireworks algorithms significantly increase the accuracy of found solutions, yet decrease the running time dramatically. At last, some applications of fireworks algorithm are briefly described, while its shortcomings and future research directions are identified.

Keywords: Fireworks Algorithm, Function Optimization, Graphic Processing Unit, Improved Fireworks Algorithm, Multi-Objective Fireworks Algorithm

1. INTRODUCTION

In most engineering fields, many problems can be simplified as numerical optimization problems through mathematical modeling. In some of the problems – not only the optimal solution, but also multiple feasible solutions

and viable localized optimal solutions need to be identified to provide enough information for decision makers. Such problems are generally referred to as multi-modal and multi-objective optimization problems. To solve those problems, the maximum or the minimum values of the functions need to be found out.

DOI: 10.4018/ijrir.2013100103

Traditional methods generally solve a continuous and differentiable function using mathematical techniques based on gradient information. However, when dealing with multi-modal and multi-objective optimization problems, traditional methods cannot always obtain even the reasonable solutions. In order to solve function optimization problems efficiently, many algorithms inspired by biological behavior are suggested recently.

The study of biological phenomena is no longer constrained in the biology discipline alone, but expanded to mathematics, computer science, information science and other research fields. Inspired by the behavior of groups of animals, many swarm intelligence algorithms are designed in the field of computer science.

Swarm can be described as a number of individuals in adjacent areas and those individuals interact with each other. In nature, a bee, or an ant, or a bird can hardly survive without its kin. A group of organics, therefore, such as the aforementioned bees, ants or birds, has more chances to survive than the lone individual. The survival chance for a group is not a simple composition of each individual's chance, but a more complex summary of social and group dynamics. The character of animal groups can greatly help its individuals adapt to their environment. Each individual obtains information from social interaction and that information gained by an individual in a group is more than the information any single individual can obtain alone. Information is then transferred among the group and each individual processes this transferred information and change its own behavior, including its own behavioral patterns and norms. Therefore, the whole group has some capabilities and characteristics, especially the ability to adapt to their environment that a single individual can hardly gain when working alone. The ability of an individual to change according with environment is known as intelligence and this intelligence is gained by the clustering of individuals.

Inspired by nature, many swarm intelligence algorithms are proposed. Observing the way of ants finding food, ant colony optimization

(ACO) algorithm was proposed by Colomi and his partners in 1991 (Colomi, Dorigo, & Maniezzo, 1991). Moreover, particle swarm optimization (PSO) algorithm was put forward by Kennedy and Eberhart (). The algorithm mimics the pattern of birds flying to find food. Yet, differential evolution (DE) algorithm is another swarm intelligence algorithm, which was given by Storn and Price (1995). In this algorithm, the differences between individuals are fully utilized. The recently announced artificial bee colony algorithm (ABC) and fish school search algorithm (FSS), were proposed in 2008 and 2009 respectively (Karaboga, & Basturk, 2008; Filho, de Lima Neto, Lins, Nascimento, & Lima, 2009). The most recently proposed fireworks algorithm (FWA) is a swarm intelligence algorithm that was published by Tan and Zhu (2010). This algorithm is inspired by fireworks explosion at night and is quite effective at finding global optimal value. As a firework explodes, a shower of sparks is shown in the adjacent area. Those sparks will explode again and generate other shows of sparks in a smaller area. Gradually, the sparks will search the whole solution space in a fine structure and focus on a small place to find the optimal solution.

As a practical optimization algorithm, fireworks algorithm can fulfill three user requirements (Storn, & Price, 1997). First of all, FWA can process linear, non-linear and multi-model test functions. Secondly, FWA can be parallelized in order to deal with complex practical problems. Thirdly, FWA has good convergence properties and can always find the global minimization.

This article completely summarizes fireworks algorithm, including the conventional fireworks algorithm, its improvements and its applications. All algorithms are tested on standard datasets. The remainder of this article is organized as follows. Section II presents the conventional fireworks algorithm. Section III to section V describes three improvements by several researchers. Some experiments are designed and the experimental results are shown in section VI. Section VII states multi-objective

fireworks algorithm and section VIII describes GPU-based fireworks algorithm. Applications of fireworks algorithm on solving optimization problems are given in section IX. In the last section X, conclusions and further research directions are drawn to enrich the research and enlarge the range of application of fireworks algorithm.

2. FIREWORKS ALGORITHM

After a firework exploded, the sparks are appeared around a location. The process of exploding can be treated as searching the neighbor area around a specific location. Inspired by fireworks in real world, fireworks algorithm (FWA) is proposed. Fireworks algorithm utilizes N D-dimensional parameter vectors x_i^G as a basic population in each generation. Parameter i varied from 1 to N and parameter G stands for the index of generations.

Every individual in the population ‘explodes’ and generates sparks around him/her. The number of sparks and the amplitude of each individual are determined by certain strategies. Furthermore, a Gaussian explosion is used to generate sparks to keep the diversity of the population. Finally, the algorithm keeps the best individual in the population and selects the rest $(N - 1)$ individuals based on distance for next generation.

More specific strategies of fireworks algorithm can be described as follows.

2.1. Explosion Sparks Strategy

The explosion sparks strategy mimics the explosion of fireworks and is the core strategy in fireworks algorithm. When a spark blasts, the spark is vanished and many sparks appear around it. The explosion sparks strategy mimicking this phenomenon is used to produce new individuals by explosion.

In this strategy, two parameters need to be determined. The first one is the number of sparks:

$$S_i = \hat{S} \cdot \frac{Y_{max} - f(x_i) + \varepsilon}{\sum_{i=1}^N (Y_{max} - f(x_i)) + \varepsilon} \quad (1)$$

In the formula, S_i represents the number of sparks generated by an individual from the population, where i varies from 1 to N . As a controlling parameter of the total number of generated sparks, \hat{S} is set as a constant. Suppose the goal is to find the minimal of a function. Variable Y_{max} stands for the worst fitness value in the current generation, while $f(x_i)$ is the fitness value for an individual x_i . The last parameter expressed as is used to prevent the denominator from becoming zero.

The second parameter in this strategy is the amplitude of sparks:

$$A_i = \hat{A} \cdot \frac{f(x_i) - Y_{min} + \varepsilon}{\sum_{i=1}^N (f(x_i) - Y_{min}) + \varepsilon} \quad (2)$$

Variable A_i gives the amplitude for an individual x_i to generate the explosion sparks and \hat{A} is a constant to control the amplitudes. The best fitness value Y_{min} is used to calculate amplitudes. In this formula, the last parameter helps to avoid the error of having the denominator being zero. If an individual is close to the boundary, the generated sparks may lie out of the feasible space. Therefore, a mapping method is used to keep sparks inside of the feasible space.

2.2. Mapping Strategy

The mapping strategy ensures all the individuals stay in the feasible space. If there are some outlying sparks from the boundary, they will be mapped to their allowable scopes:

$$x_i = x_{min} + |x_i| \% (x_{max} - x_{min}) \quad (3)$$

where x_i represents the positions of any sparks that lie out of bounds, while x_{max} and x_{min} stand for the maximum and minimum boundary of a spark position. The symbol % stands for the modular arithmetic operation. Aside from the explosion sparks strategy, another way to generate sparks is proposed as Gaussian sparks strategy.

2.3. Gaussian Sparks Strategy

To keep the diversity of the population, Gaussian sparks strategy is used to generate sparks with Gaussian distribution. Suppose the position of current individual is stated as x_k^j , the Gaussian explosion sparks are calculated as:

$$x_k^j = x_k^j \cdot g \quad (4)$$

where g is a random number in Gaussian distribution:

$$g = \text{Gaussian}(1,1) \quad (5)$$

Parameter g obeys the Gaussian distribution with both mean value and standard deviation are 1. After normal explosions and Gaussian explosions, we consider a proper way to select individuals for next generation. Here, a distance based selection method is suggested.

2.4. Selection Strategy

To select the individuals for next generation, the best individual is always kept at first. Then the next $(N - 1)$ individuals are selected based on their distance to other individuals. The individual that is far from other individuals gets more chance to be selected than those individuals with smaller distances to other individuals.

The general distance between two locations is calculated by:

$$R(x_i) = \sum_{j \in K} d(x_i, x_j) = \sum_{j \in K} \|x_i - x_j\| \quad (6)$$

where location x_i and x_j ($i \neq j$) can be any locations and K is the set of all current locations. For the distance measurements, many methods can be used, including Euclidean distance, Manhattan distance and Angle-based distance. Inspired by the immune density (Lu, Tan, & Zhao, 2002), Euclidean distance is used in the fireworks algorithm (Tan, & Zhu, 2010):

$$d(x_i, x_j) = |f(x_i) - f(x_j)| \quad (7)$$

where $f(x_i)$ is the fitness for location x_i and $d(x_i, x_j)$ represents the distance between two locations.

As last, a roulette wheel method is used to calculate the possibility of selecting the locations:

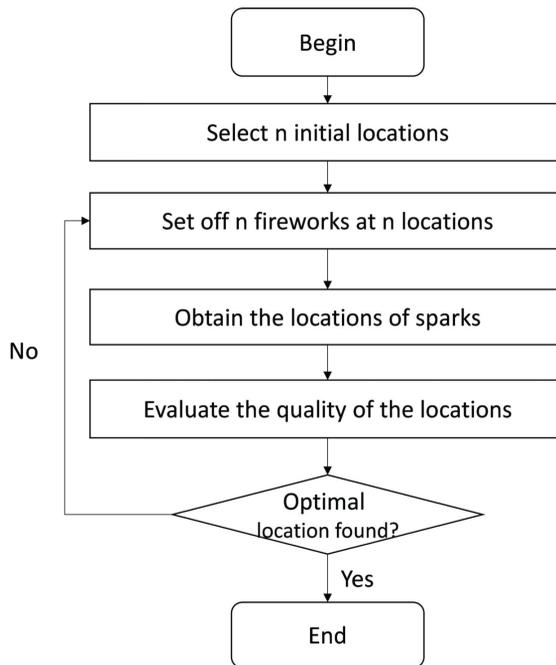
$$p(x_i) = \frac{R(x_i)}{\sum_{j \in K} R(x_j)} \quad (8)$$

The individuals with larger distance from others have more chance to be selected. In this way, the diversity of a population can be ensured.

The flowchart and pseudo code for fireworks algorithm is stated in Figure 1.

Fireworks algorithm works quite well on following parameters, where $n = 5$, $m = 50$, $a = 0.04$, $b = 0.8$, $\hat{A} = 40$ and $\hat{m} = 5$. Although fireworks algorithm reaches great progress at several problems, there are still some places for improvement. Zheng et al. (Zheng, Janecek, & Tan, 2013) proposed an enhanced fireworks algorithm, which significantly increased the accuracy of result on test functions. Liu et al. (2013) studied the exploration and exploitation abilities of fireworks algorithm and then designed a transfer function to calculate the

Figure 1. The flowchart of fireworks algorithm



number and the amplitude for sparks (see Algorithm 1). Pei et al. (2012) presented an empirical study on the influence of fitting methods of fireworks algorithm. Other related references including but not limited to Zheng, and Tan (2013), Zhou, and Tan (2009), Zhou, and Tan (2011), Bureerat (2011), Lou, Li, Jin, and Li (2012), Lou, Li, Shi, and Jin (2013) and Tan and Xiao (2007).

3. ENHANCED FIREWORKS ALGORITHM (EFWA)

To overcome the disadvantages of fireworks algorithm, many researchers have attempted in different ways to improve it. Zheng et al. (2013) proposed an enhanced fireworks algorithm through improvements it in the following five aspects.

3.1. Minimal Explosion Amplitude Setting

In the evolution process, some explosion amplitude may be close to zero, which is not conducive to find the global best value. Since the explosion amplitude was closely related to fitness values, two ways to limit the minimum amplitude boundary were proposed. One way is based on a linear function and the other is based on a non-linear function:

$$A_{\min}^k(t) = A_{\text{init}} - \frac{A_{\text{init}} - A_{\text{final}}}{\text{evals_max}} * t \quad (9)$$

$$A_{\min}^k(t) = A_{\text{init}} - \frac{A_{\text{init}} - A_{\text{final}}}{\text{evals_max}} * \sqrt{(2 * \text{evals_max} - t) * t} \quad (10)$$

Algorithm 1.

```

for (i = 0; i < N; i++)
randomly generate an individual.
while (count < gen_max) {
// gen_max is the maximum number of generation
// For each individual, generate Si sparks within amplitude Ai.
for (i = 0; i < N; i++) {
Set  $z^k = \text{round}(\text{rand}(0, 1))$ ,  $k = 1, 2, \dots$ , dimension
For each dimension of  $x_i^k$ 
if ( $z^k == 1$ )  $x_i^k += A_i * \text{rand}(-1, 1)$ ;
if ( $x_i^k$  is out of scope) execute mapping operation;
calculate fitness;
evaluation count += Si;
}
// Generate Gaussian Sparks
for (i = 0; i <  $\hat{m}$ ; i++) {
Set  $z^k = \text{round}(\text{rand}(0, 1))$ ,  $k = 1, 2, \dots$ , dimension
For each dimension of  $x_i^k$ 
if ( $z^k == 1$ )  $x_i^k * = \text{Gaussian}(1, 1)$ ;
if ( $x_i^k$  is out of bounds) execute mapping operation;
calculate fitness;
evaluation count +=  $\hat{m}$ ;
}
//Selection
Keep the best individual for next generation
Then select (N - 1) individuals
if(evaluation count > evals_max) break;
//evals_max is the maximum number of evaluations
}

```

In both formulae, $A_{min}^k(t)$ means the lower boundary for an individual in the k dimension when the function is evaluated t times. The two new parameters A_{init} and A_{final} stands for the initiate and final amplitudes. The last parameter is the maximum evaluation times, which is expressed as $evals_max$. The schematic diagrams for linear and non-linear minimal explosion amplitudes are drawn in Figure 2.

3.2. Explosion Sparks Strategy

In the fireworks algorithm, the same increment will be added to some selected dimensions of an individual.

As it is shown in Figure 4, the same increment may cause a loss of diversity to a popula-

tion. Hence, it is necessary to generate different increments and add the increments to each selected dimension for an individual to obtain the diversity of population. In Figure 3, $x_i^j (j = 1, 2, \dots, Dimension)$ stands for the value in the jth dimension of the ith individual. A_i is the amplitude for that individual x_i .

3.3. Gaussian Sparks Strategy

The fireworks algorithm works significantly well on functions that will reach their optimal at the origin of coordinate. For example, the optimal value of a two-dimensional Ackley function lies at the origin of its coordinate. But if the function is shifted, e.g. the optimal value is shifted to [-70, -55], the fireworks algorithm performs badly. Figure 4 shows the location of

Figure 2. The schematic diagrams of minimal amplitude for linear and non-linear decreases. This figure is obtained from the work of (Zheng, Janecek, & Tan, 2013).

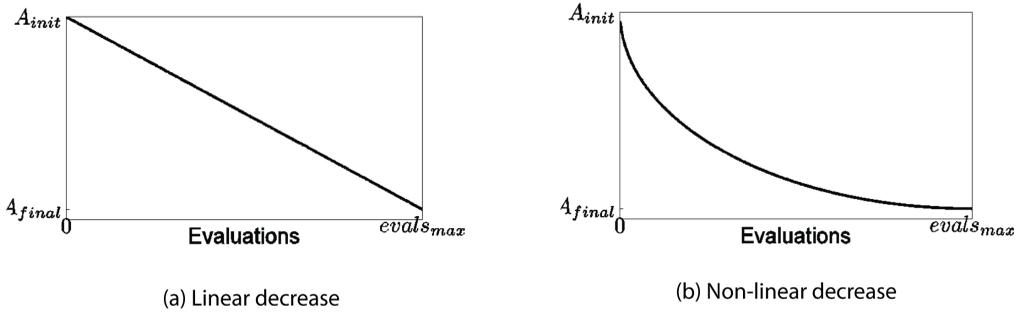


Figure 3. Increment of FWA and EFWA in each selected dimension

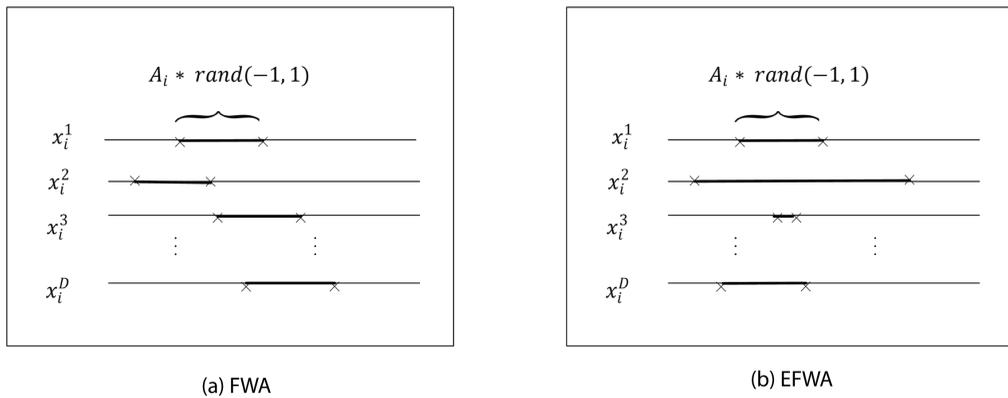
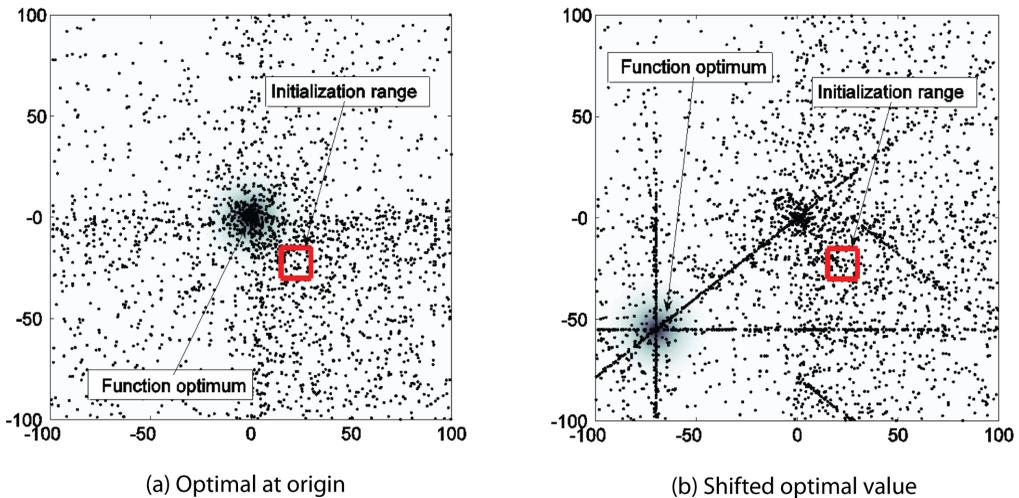


Figure 4. Effect of the Gaussian sparks. This figure comes from the work of (Zheng, Janecek, & Tan, 2013).



the Gaussian sparks in fireworks algorithm. It can be seen that Gaussian sparks can easily find the optimal value at the origin of coordinate when the function is not shifted. But Gaussian sparks work poorly on the shifted function.

To overcome the disadvantage of Gaussian sparks, Zheng et al. (2013) used another way to generate Gaussian sparks. Referring the position of the current global best individual, the Gaussian sparks are generated by:

$$x_i^k = x_i^k + (x_{Best}^k - x_i^k) * g \quad (11)$$

where g is a random number obeyed Gaussian distribution, i.e.:

$$g = \text{Gaussian}(0,1) \quad (12)$$

In the formula, x_i^k stands for the selected individual to generate Gaussian sparks and x_{Best}^k is the best individual the algorithm has find out so far. Parameter g obeys the Gaussian distribution of mean 0 and standard deviation 1.

3.4. Mapping Strategy

The proposed fireworks algorithm used modular arithmetic operation to map individuals back into scope. However, modular arithmetic operation is time consuming. Besides, some of the individuals are mapped to a place near the origin, straying from the diversity of population. For example, suppose the solution space varies from -20 to 20. If there is an individual who has a value of -21, then it maps to 1 according to the formula suggested in fireworks algorithm. Hence, a new mapping operator is proposed:

$$x_i^k = x_{min}^k + \text{rand}(0,1) * (x_{max}^k - x_{min}^k) \quad (13)$$

where x_{min}^k and x_{max}^k are the lower and upper boundary of the solution space.

3.5. Selection Strategy

The most time consuming part of conventional fireworks algorithm lies in the selection. In the selection strategy of conventional fireworks algorithm, the distances between individuals need to be calculated. Hence, the computational complexity of selection strategy is much higher than random selection strategy.

The selection operation is called as Elitism Random Selection (ERS). According to the work of Pei et al. (2012), the best individual is always preserved for next generation, while the other $(N - 1)$ individuals are selected randomly. In this way, the running time for fireworks algorithm is largely decreased and furthermore, the computational complexity is linear.

4. IMPROVED FIREWORKS ALGORITHM (IFWA) WITH TWO DIFFERENT SELECTION METHODS

Liu et al. (2013) put forward another effective improvement of fireworks algorithm. The individuals are sorted by their fitness values in increasing order and two new formulae are given concerning the number and the amplitude of sparks:

$$S_i = \hat{S} \cdot \frac{t(i)}{\sum_{i=1}^N t(i)} \quad (14)$$

$$A_i = \hat{A} \cdot \frac{t(N - i + 1)}{\sum_{i=1}^N t(N - i + 1)} \quad (15)$$

where $t(i)$ is a transfer function with a:

$$t(i) = \frac{1}{1 + e^{-\frac{(i-1)}{a}}} \quad (16)$$

Transfer function helps to decrease the number and the amplitude of sparks evenly. Parameter varies from 20 to 1 with an even number of distributions for each generated value. \hat{S} and \hat{A} are set as constant parameters controlling of the total number and the maximum amplitude of sparks respectively. N stands for the number of individuals, while S_i and A_i are the number and the scope for an individual x_i to generate the explosion sparks.

A random function is proposed to replace the function to generate Gaussian sparks:

$$x_i^k = x_{min}^k + (x_{max}^k - x_{min}^k) * rand(0,1) \quad (17)$$

The function randomly generates individuals and ensures the generated individuals are in the feasible space. x_{min}^k and x_{max}^k are the maximum and minimum boundaries of the k^{th} dimension.

Two different selection methods are proposed, named as the best fitness selection and roulette fitness selection:

1. **Best Fitness Selection (BFS):** The best fitness selection is first proposed by Zheng et al. (2013). The best individuals are selected for next generation from both the basic individuals and the generated sparks;
2. **Roulette Fitness Selection (RFS):** After the algorithm selects the best individual, other individuals are selected by roulette based on their fitness values. The probability for each individual to be selected is calculated as:

$$p(x_i) = \frac{Y_{max} - f(x_i)}{\sum_{i=1}^K (Y_{max} - f(x_i))} \quad (18)$$

where Y_{max} stands for the worst fitness value in the population, while $f(x_i)$ is the fitness for an individual x_i . Parameter K means the total number of individuals, including basic individuals, explosion sparks and Gaussian

sparks. It can be seen that the individuals with lower fitness value have more chance to be selected.

5. THE INFLUENCE OF APPROXIMATION APPROACHES ON ENHANCED FIREWORKS ALGORITHM

To figure out the influence of sampling methods and fitting methods on enhanced fireworks algorithm, Pei et al. (2012) proposed three sampling methods and two fitting methods in enhanced fireworks algorithm. Also, they used a random selection method to choose individual for next generation.

5.1. Sampling Methods

1. **Best Sampling:** The best K individuals are selected as sampling data;
2. **Distance near the Best Fitness Individual Sampling:** By calculating the Euclidean distance between the best individual and the other individuals, the nearest K individuals are selected as sampling data;
3. **Random Sampling:** K individuals are selected randomly as sampling data.

5.2. Fitting Methods

In order to generate a new spark, three sampling methods and two fitting methods are used and compared. The first sampling method is to select the best K individuals (BST), whereas K can be defined as 3, 5 or 10. The second sampling method is to pick up the K individuals, which have the smallest distance from the best individual (DIS). The third sampling method is to choose the K individuals randomly (RAN). The two fitting methods are linear least square approximation (LS1) and non-linear two degree polynomial approximation (LS2).

Three sampling method are tested with two fitting method. Hence, six different methods are proposed. For example, LS1-BST3 means

to select the best three individuals among the population and generates a new spark from these selected individuals using linear least square approximation method. In each dimension, a line segment generates and the value of the middle point is taken as the new spark. As for non-linear fitting methods, the value of the extreme point is treated as the new spark. The new spark replaces the worst individual in the population if the new spark is better.

5.3. Selection Methods

In the paper Pei, Zheng, Tan, and Takagi (2012), Pei et al. keeps the best individual for next generation and randomly selects the rest $(N - 1)$ individuals. The selection method is named as Random Selection with Replacement (RSR).

6. EXPERIMENTS

6.1. Design of Experiments

Thirteen test functions are chosen to verify the performance of conventional fireworks algorithm, its variants and standard particle

swarm optimization as seen in Table 1 (Bratton, & Kennedy, 2007).

According to the work of Zheng et al. (2013), fireworks algorithm works extremely well on those functions whose optimum is located at original point $(0, 0)$, because the Gaussian sparks can easily find that point. To shift the global optimal value away from point $(0, 0)$, a number of shifted values are added to the functions. Here, the optimum of the functions is shifted to the right corner of the feasible search space. Table 2 shows the shifted values.

X_{max} and X_{min} means the maximum and minimum boundaries for the individual, respectively. Still, there are two more parameters in EFWA named as A_{init} and A_{final} . The two parameters are set as $(X_{max} - X_{min}) * 0.02$ and $(X_{max} - X_{min}) * 0.001$, respectively.

The experimental platform is Visual Studio 2012 and the program is running on 64-bit Window 8 operation system with an Intel Core i7-3820QM with 2.70GHz and 2GBRAM. Each experiment runs 30 times and during each run, the fitness functions are evaluated just over 300,000 times. The function evaluation cannot

Table 1. The details of benchmark functions

No.	Name	Attributes	Optimization Value	Initial Population	Dimension
F1	Sphere	Unimodal	0.0	$[-100, 100]^D$	30
F2	Schwefel's Problem 1.2	Unimodal	0.0	$[-100, 100]^D$	30
F3	Generalized Rosenbrock	Unimodal	0.0	$[-30, 30]^D$	30
F4	Ackley	Unimodal	0.0	$[-32, 32]^D$	30
F5	Generalized Griewank	Unimodal	0.0	$[-600, 600]^D$	30
F6	Penalized Function F8	Multimodal	0.0	$[-50, 50]^D$	30
F7	Penalized Function P16	Multimodal	0.0	$[-50, 50]^D$	30
F8	Six-hump Camel Back	Multimodal	-1.032	$[-5, 5]^D$	2
F9	Goldstein-Price	Multimodal	3.0	$[-2, 2]^D$	2
F10	Schaffer F6	Multimodal	0	$[-100, 100]^D$	2
F11	Axis Parallel Hyper Ellipsoid	Multimodal	0	$[-5.12, 5.12]^D$	30
F12	Rotated Hyper Ellipsoid	Multimodal	0	$[-65.536, 65.536]^D$	30
F13	Generalized Rastrigin	Multimodal	0	$[-5.12, 5.12]^D$	30

Table 2. Shifted index and shifted value (index zero means no shift)

Shifted Index	Shifted Value
0	0
1	$0.025 * (X_{max} - X_{min})$
2	$0.050 * (X_{max} - X_{min})$
3	$0.100 * (X_{max} - X_{min})$
4	$0.150 * (X_{max} - X_{min})$
5	$0.250 * (X_{max} - X_{min})$
6	$0.350 * (X_{max} - X_{min})$

be equal to 300,000 because the number of sparks is not fixed in each generation. Therefore, once the number of function evaluations exceeds 300,000 at the end of a generation, there will not be any further generations.

According to reference Bratton and Kennedy (2007), standard particle swarm optimization (SPSO) includes a ring topology when the particles are only communicated with their two neighbors. Moreover, the number of particles is set as 50, while the initialization is non-uniform and the evaluation operations are skipped if the particles are out of the feasible search space.

6.2. Experimental Results

Six swarm intelligence algorithms are compared, including the conventional fireworks algorithm, four improved fireworks algorithms and the SPSO algorithm.

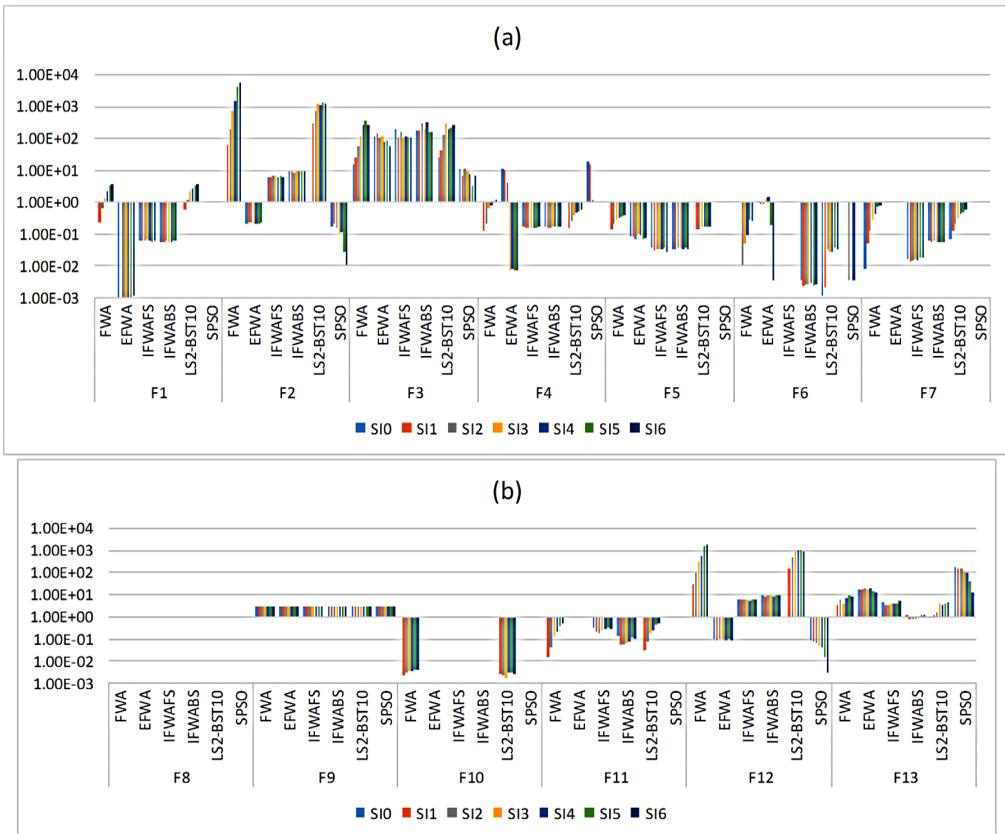
The parameters of SPSO algorithm are the same as in reference Bratton et al (2007), while the other four improved fireworks algorithms have the same parameters with the conventional algorithm in reference (Tan, & Zhu, 2010). EFWA is proposed by Zheng (Zheng, Janecek,

& Tan, 2013), whereas improved fireworks algorithm with fitness value selection (IFWAFS) and improved fireworks algorithm with best selection (IFWABS) can be found in reference Liu, Zheng, and Tan (2013). The algorithm named LS2-BST10 is the best algorithms stated in article Pei, Zheng, Tan, and Takagi (2012) with extinguished sampling numbers and fitting methods. LS2-BST10 means the sampling method is non-linear and the best ten individuals are selected.

In order to make the figures more easily readable, the experiment results are divided into two figures, namely Figure 5(a) and Figure 5(b). Thus, the horizontal and vertical axis in Figure 5(a) have the same meaning as Figure 5(b). The horizontal axis stands for the six algorithms along with 13 functions and the vertical axis represents the mean values in the form of logarithm. Some bar figures are not shown because the corresponding mean values are below zero and the logarithm operation cannot be performed.

Figure 6 depicts the running time for each algorithm on the 13 functions. The vertical axes

Figure 5. (a) Mean values of conventional fireworks algorithm and its variants on function 1 to 7 (b) Mean values of conventional fireworks algorithm and its variants on function 8 to 13



in both Figure 6 (a) and Figure 6 (b) represent the running time in seconds. The higher the bar graph is, the more time consuming the algorithm for each function.

Table 3 represents the mean values and the standard deviations of each algorithm running on all the functions with shifted indexes from 0 to 6.

Table 4 shows the t-test results of fireworks algorithm against every other algorithms. The t-test results of fireworks algorithm with the other algorithms are shown below. The bold values in the table indicate the other algorithms are significant better than FWA. If the experiment results are the same, there is no t-test result and a sign NAN is shown.

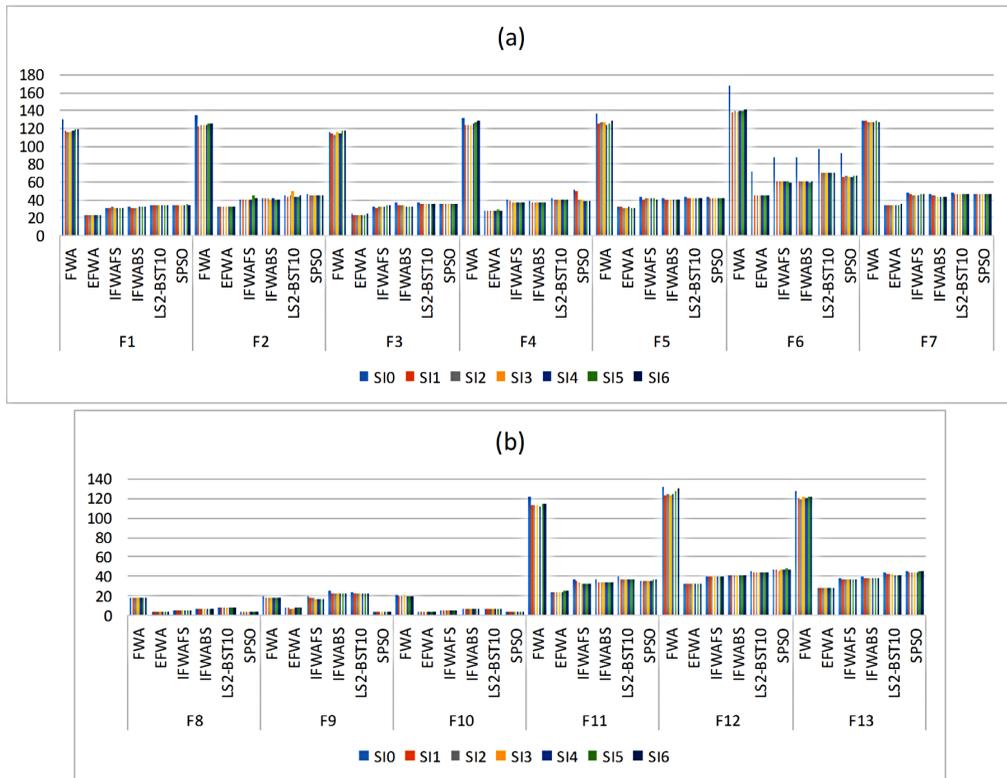
The running time of each algorithm is shown in Table 5. The unit of the results is second.

6.3. Discussions

The following observations can be concluded from the experimental results in Table 5:

1. EFWA, IFWAFS, IFWABS and LS2-BST10 are superior to conventional FWA on most functions;
2. With increasing shifted values, EFWA achieves much better results than conventional FWA;
3. EFWA performs steadily even the optimum is shifted to the edge of the feasible space;

Figure 6. (a) Running time of conventional fireworks algorithm and its variants on function 1 to 7 (b) Running time of the same algorithms on function 8 to 13



4. SPSO achieves better results on large shifted indexes;
5. Improved fireworks algorithms, including EFWA, IFWAFS, IFWABS and LS2-BST10, are worse than SPSO on most functions;
6. EFWA is extremely fast on 11 functions, while SPSO is quicker than other algorithms on 2 other functions;
7. Conventional FWA consumes much more time than all the other algorithms.

7. MULTI-OBJECTIVE FWA

Conventional FWA and its variants can solve problems with single objective. When dealing

with multi-objective optimization problems, those algorithms are no longer useful and efficient.

Zheng et al. (2013) firstly studied multi-objective fireworks algorithm (MOFOA) and applied it to optimize variable-rate fertilization in oil crop production. Three objectives were chosen and the data from three oil crops were used. The distribution of solutions by MOFOA was given and it was also compared with multi-objective random search (MORS).

Fertilizing Oil Crops can be described as a multi-objective problem with three objectives. They are crops quality, fertilizer cost and energy consumption. Compare with conventional FWA, two new strategies were proposed.

Table 3. Mean and standard deviation of FWA and other algorithms with shifted values

		S	I	0	S	I	1	S	I	2	S	I	3	S	I	4	S	I	5	S	I	6	
F1	FWA	0.00e+000(0.0e+000)	2.489e+022(7.1e+001)		1.001e+003(2e+002)		4.167e+001(3e+003)		7.742e+002(2.9e+003)		1.322e+004(4e+003)		1.242e+006(6e+003)										
	FWA	1.059e+004(2e+004)	9.603e+004(2e+004)		1.041e+003(3e+004)		1.049e+003(3e+004)		1.048e+003(3e+004)		1.079e+003(3e+004)		1.079e+003(3e+004)										
	FWF	5.889e+002(2e+002)	5.744e+002(1.7e+002)		5.961e+002(2e+002)		5.461e+002(1e+002)		5.424e+002(2e+002)		6.079e+002(1e+002)		6.411e+002(2e+002)										
	FWP	0.00e+000(0.0e+000)	5.613e+001(2.2e+001)		1.158e+000(6e+001)		2.279e+000(7.8e+001)		2.694e+000(1.3e+000)		3.414e+000(1.3e+000)		3.830e+000(1.3e+000)										
	SFSO	3.117e+024(3e+024)	0.00e+000(0.0e+000)																				
F2	FWA	0.00e+000(0.0e+000)	1.353e+004(1e+002)		4.064e+001(1e+003)		9.934e+002(2e+003)		1.41.5e+004(6e+003)		1.889e+003(4e+003)		4.00e+004(6e+004)										
	FWA	2.181e+001(5e+002)	2.257e+001(8.2e+002)		2.311e+001(8.4e+002)		2.195e+001(7.5e+002)		2.075e+001(3.4e+002)		2.121e+001(7.8e+002)		2.321e+001(7.9e+002)										
	FWF	0.308e+002(1e+000)	0.175e+001(5e+000)		7.016e+002(5e+000)		6.705e+002(3e+000)		6.059e+001(5e+000)		6.811e+001(6e+000)		5.933e+002(1e+000)										
	FWP	6.301e+002(2e+000)	9.453e+002(2e+000)		8.833e+002(5e+000)		9.485e+002(5e+000)		9.414e+002(2e+000)		8.975e+002(2e+000)		8.754e+002(2e+000)										
	SFSO	0.00e+000(0.0e+000)	2.952e+002(5e+001)		7.263e+002(1e+002)		1.186e+003(1e+002)		1.123e+003(8e+002)		1.391e+003(3e+002)		1.242e+003(8e+002)										
F3	FWA	2.84e+001(0.8e+001)	7.872e+003(3.1e+003)		9.033e+004(1e+004)		1.256e+003(1e+005)		4.552e+001(6e+006)		9.359e+006(6e+006)		2.311e+002(2e+007)										
	FWA	1.049e+001(4e+002)	1.482e+002(1.6e+002)		1.020e+002(1e+002)		1.178e+002(3e+002)		7.906e+001(7.0e+001)		8.149e+001(8.0e+001)		5.529e+001(3.2e+001)										
	FWF	0.308e+002(2e+000)	1.102e+002(8.1e+001)		1.559e+002(7e+002)		1.020e+002(5e+001)		1.175e+002(4.3e+001)		1.049e+002(4e+001)		1.024e+002(2e+001)										
	FWP	1.75e+002(8.1e+001)	1.784e+002(8.1e+001)		2.907e+002(3e+002)		2.025e+002(4e+002)		3.554e+002(1e+002)		1.453e+002(4e+001)		1.511e+002(6.4e+001)										
	SFSO	2.199e+002(2e+001)	4.308e+001(1.9e+001)		1.131e+003(1.5e+002)		3.101e+002(3e+002)		1.025e+002(5e+001)		3.233e+002(7e+001)		2.531e+002(6e+001)										
F4	FWA	0.00e+000(0.0e+000)	4.725e+000(4e+000)		8.632e+000(5e+000)		1.388e+001(2e+000)		2.545e+001(7e+000)		1.979e+001(1e+000)		1.928e+001(2e+001)										
	FWA	1.171e+001(0.4e+000)	1.053e+001(0.6e+000)		3.996e+000(6e+000)		7.016e+001(1e+003)		7.816e+001(1e+003)		7.509e+001(1e+003)		7.509e+001(1e+003)										
	FWF	1.005e+001(0e+002)	1.648e+002(2.8e+002)		1.641e+002(3e+002)		1.572e+001(2.7e+002)		1.698e+002(1e+002)		1.801e+002(1.2e+002)		1.652e+002(2e+002)										
	FWP	1.751e+001(3e+002)	1.021e+001(3.4e+002)		1.659e+001(0.6e+002)		1.691e+001(3.2e+002)		1.051e+001(3.4e+002)		1.895e+001(3.1e+002)		1.711e+001(3.1e+002)										
	SFSO	0.00e+000(0.0e+000)	1.879e+001(2.5e+001)		2.442e+001(3e+001)		4.117e+001(1.8e+001)		4.759e+001(6e+001)		5.384e+001(1.1e+001)		5.934e+001(1.5e+001)										
F5	FWA	0.00e+000(0.0e+000)	3.596e+000(7e+001)		1.124e+002(5e+000)		3.631e+001(1.1e+001)		8.005e+001(2.6e+001)		1.025e+001(5e+001)		3.828e+001(5e+001)										
	FWA	8.848e+002(3.2e+002)	8.392e+003(2e+002)		7.263e+004(6e+002)		1.031e+001(6e+002)		9.132e+002(8e+002)		6.001e+002(2.5e+002)		7.520e+002(5e+002)										
	FWF	3.063e+002(2e+002)	3.117e+002(2.2e+002)		3.373e+002(6e+002)		3.446e+002(2.5e+002)		3.486e+002(2.5e+002)		3.486e+002(2.5e+002)		2.970e+002(5e+002)										
	FWP	3.086e+002(5e+002)	3.286e+001(8e+002)		3.855e+002(2e+002)		3.844e+002(2.5e+002)		3.548e+002(2.5e+002)		3.916e+002(3e+002)		3.429e+002(2e+002)										
	SFSO	0.00e+000(0.0e+000)	1.374e+001(4e+002)		1.351e+002(1e+002)		1.751e+001(5.4e+002)		1.944e+001(4.4e+002)		1.846e+001(6.1e+002)		1.824e+001(5e+002)										
F6	FWA	2.644e+001(1e+001)	0.621e+001(4.8e+001)		3.111e+000(5e+002)		3.556e+001(8.3e+001)		2.331e+001(2e+005)		6.489e+006(2e+006)		2.561e+001(6e+007)										
	FWA	1.142e+001(1e+000)	9.272e+001(1.6e+000)		9.011e+001(1.5e+000)		1.341e+001(7e+000)		1.499e+001(9e+000)		1.804e+001(1e+000)		3.462e+001(3e+000)										
	FWF	8.11e+001(0.4e+004)	7.089e+002(7e+004)		7.014e+002(7e+004)		7.301e+002(7e+004)		7.817e+002(7e+004)		9.14e+002(7e+004)		8.795e+002(7e+004)										
	FWP	3.333e+001(9e+003)	2.451e+003(8.3e+004)		2.555e+001(3e+003)		2.680e+001(3e+003)		2.971e+001(3e+003)		2.975e+001(3e+003)		2.975e+001(3e+003)										
	SFSO	1.119e+001(1e+003)	2.076e+002(2e+000)		3.506e+002(1e+002)		2.886e+002(3e+002)		2.727e+002(2e+002)		3.874e+002(2e+002)		3.538e+002(1e+002)										
F7	FWA	1.371e+003(3e+004)	1.571e+003(3e+004)		1.456e+001(9e+003)		1.571e+003(3e+004)		1.856e+003(1e+003)		1.371e+003(3e+004)		1.571e+003(3e+004)										
	FWA	2.888e+001(3e+001)	1.883e+001(5.2e+001)		1.023e+002(6e+001)		3.278e+001(5e+005)		1.108e+001(7e+006)		3.613e+001(2e+007)		7.482e+001(8e+007)										
	FWA	3.924e+001(3e+005)	3.878e+002(2e+005)		4.077e+001(5e+005)		3.725e+001(7e+005)		4.275e+001(9e+005)		3.826e+001(3e+005)		4.373e+001(2e+005)										
	FWF	1.142e+002(1e+005)	1.429e+003(2e+005)		1.549e+003(2e+005)		1.637e+002(1e+003)		1.257e+002(1e+003)		1.842e+002(1e+003)		1.842e+002(1e+003)										
	SFSO	6.019e+002(2e+002)	5.743e+001(9e+002)		6.027e+001(8e+002)		5.805e+002(2e+002)		5.700e+001(8e+002)		5.893e+001(6e+002)		5.691e+002(2e+002)										
F8	FWA	1.118e+004(4e+002)	1.314e+001(4.7e+002)		2.079e+001(2e+002)		3.086e+001(9e+002)		4.268e+001(1e+003)		4.899e+001(2e+003)		4.899e+001(2e+003)										
	FWA	1.359e+002(5e+004)	1.359e+002(5e+004)																				
	FWF	-1.032e+003(1e+007)	-1.032e+003(1e+007)																				
	FWP	-1.032e+003(1e+007)	-1.032e+003(1e+007)																				
	SFSO	-1.032e+003(1e+007)	-1.032e+003(1e+007)																				
F9	FWA	3.000e+000(1e+004)	3.000e+000(1e+004)		3.000e+000(3e+004)																		
	FWA	3.000e+000(1e+000)	3.000e+000(1e+000)																				
	FWF	3.000e+000(1e+000)	3.000e+000(1e+000)																				
	FWP	3.000e+000(1e+000)	3.000e+000(1e+000)																				
	SFSO	3.000e+000(1e+000)	3.000e+000(1e+000)																				
F10	FWA	0.00e+000(0.0e+000)	4.575e+001(1.3e+001)		4.375e+001(1.3e+001)		4.575e+001(1.3e+001)		4.75e+001(1.3e+001)		4.75e+001(1.3e+001)		4.75e+001(1.3e+001)										
	FWA	8.731e+001(8.7e+007)	7.524e+001(7.5e+007)		1.024e+001(4e+009)		1.231e+002(2e+009)		1.064e+001(3e+009)		9.232e+001(1e+009)		1.024e+001(4e+009)										
	FWF	3.653e+001(5e+007)	2.669e+001(5e+007)																				

Table 4. T-test results for FWA VS other algorithms

		S I 0	S I 1	S I 2	S I 3	S I 4	S I 5	S I 6
F1	FFWA	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
	FFWS	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
	FFWS	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
	LSM	NaN	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
F2	FFWA	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.00012121
	FFWS	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.00012143
	FFWS	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.00012155
	LSM	NaN	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.00018158
F3	FFWA	0.001698719	0.000000000	0.000000000	0.000000000	0.000000000	0.000000014	0.000006926
	FFWS	0.009688508	0.000000000	0.000000000	0.000000000	0.000000000	0.000000014	0.000006927
	FFWS	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000014	0.000006927
	LSM	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000014	0.000006926
F4	FFWA	0.000000313	0.002853052	0.002922802	0.000000000	0.000000000	0.000000000	0.000000000
	FFWS	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
	FFWS	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
	LSM	NaN	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
F5	FFWA	0.000000000	0.000000264	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
	FFWS	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000002
	FFWS	0.000000029	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000002
	LSM	NaN	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000002
F6	FFWA	0.325581988	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000002
	FFWS	0.008562417	0.392489849	0.000000007	0.000000000	0.000000000	0.000001636	0.000657971
	FFWS	0.000000007	0.000000039	0.000000000	0.000000000	0.000000000	0.000001636	0.000657971
	LSM	0.000000008	0.000000041	0.000000000	0.000000000	0.000000000	0.000001636	0.000657971
F7	FFWA	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000101	0.000084571
	FFWS	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000101	0.000084571
	FFWS	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000101	0.000084571
	LSM	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000101	0.000084571
F8	FFWA	0.000244879	0.000615038	0.021836460	0.002381466	0.003457959	0.000049865	0.004188248
	FFWS	0.106555198	0.001001718	0.029754181	0.002910600	0.003594118	0.000052123	0.004222141
	FFWS	0.001191246	0.000707112	0.022263809	0.002418769	0.003473945	0.000050402	0.004196431
	LSM	0.001124488	0.001195725	0.028853886	0.002698376	0.003687445	0.000054462	0.004320319
F9	FFWA	0.000244879	0.000615038	0.021836460	0.002381466	0.003457959	0.000049865	0.004188248
	FFWS	0.000244879	0.000615038	0.021836460	0.002381466	0.003457959	0.000049865	0.004188248
	FFWS	0.004629525	0.000118236	0.000076419	0.000104555	0.007461993	0.001120415	0.001433219
	LSM	0.010355369	0.000530539	0.000130786	0.000138918	0.007975965	0.002980494	0.001542341
F10	FFWA	0.005412157	0.000174996	0.000081426	0.000124419	0.007639982	0.001500402	0.001491748
	FFWS	0.004692955	0.000202631	0.000091841	0.000129511	0.007876122	0.001154433	0.001587999
	FFWS	0.004629505	0.000118234	0.000076418	0.000104555	0.007461988	0.001120380	0.001433214
	LSM	0.0000006171	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000184
F11	FFWA	0.003291062	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000184
	FFWS	0.000440739	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000184
	FFWS	NaN	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000184
	LSM	0.043384982	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000184
F12	FFWA	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000165
	FFWS	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000162
	FFWS	NaN	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000168
	LSM	NaN	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000161
F13	FFWA	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
	FFWS	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
	FFWS	NaN	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
	LSM	NaN	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
F14	FFWA	0.000000105	0.000004135	0.000033876	0.000009171	0.000037778	0.000031965	0.026562953
	FFWS	0.000000000	0.000000002	0.000000188	0.000000000	0.000000000	0.000000000	0.000000000
	FFWS	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
	LSM	NaN	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
F15	FFWA	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
	FFWS	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
	FFWS	NaN	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
	LSM	NaN	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000

Table 5. Time consuming for each algorithm

		S I 0	S I 1	S I 2	S I 3	S I 4	S I 5	S I 6	
F1	FWA	115.660	113.841	113.702	113.906	113.906	114.503	114.484	
	EFWA	22.689	22.536	22.517	22.504	22.519	22.579	22.535	
	FWIS	30.301	30.300	31.734	30.721	31.170	30.158	30.175	
	FWIS	32.130	31.661	31.646	31.660	32.052	32.208	32.519	
	USATO	34.296	33.567	33.629	33.692	33.692	33.707	34.098	
SPSO	FWA	33.926	34.268	34.270	34.426	34.785	34.893	34.738	
	F2	FWA	127.001	125.656	125.687	127.326	126.624	125.608	125.576
	EFWA	32.426	32.457	32.411	32.426	32.471	32.396	32.410	
	FWIS	40.177	40.205	40.288	40.098	40.739	44.632	41.598	
	FWIS	42.323	41.224	41.443	41.082	41.740	41.129	41.098	
USATO	FWA	44.740	44.160	45.295	49.780	44.021	44.101	44.694	
	FWIS	46.178	45.648	45.662	45.817	45.115	44.959	44.584	
	SPSO	FWA	119.813	118.248	118.140	115.780	115.625	115.610	116.140
	EFWA	24.456	23.658	23.769	23.661	23.785	23.691	23.895	
	FWIS	33.191	31.224	32.174	31.715	32.813	33.394	33.443	
FWIS	FWA	36.647	34.207	33.864	33.567	33.207	32.942	32.661	
	FWIS	37.490	35.396	35.443	35.489	35.474	35.708	35.990	
	SPSO	35.035	35.019	34.927	35.487	35.319	35.112	35.395	
	F4	FWA	121.103	121.170	120.326	120.373	120.857	120.546	120.311
	EFWA	28.483	27.768	27.692	27.627	27.911	28.740	28.087	
FWIS	FWA	40.830	38.271	37.543	37.501	37.541	37.829	37.641	
	FWIS	39.129	36.818	36.863	36.864	36.818	36.895	36.866	
	USATO	41.224	39.787	39.817	39.818	39.896	40.193	40.477	
	SPSO	52.037	49.100	40.959	39.661	39.225	39.363	39.426	
	F5	FWA	124.123	123.359	123.374	123.452	123.281	123.405	123.518
EFWA	FWA	32.671	32.082	31.638	31.582	31.941	31.300	31.254	
	FWIS	43.386	41.044	41.825	41.371	41.183	41.476	41.144	
	FWIS	42.271	40.056	40.097	39.989	39.976	40.021	40.177	
	USATO	44.208	42.349	42.272	42.271	42.317	42.366	42.380	
	SPSO	42.817	41.740	41.614	42.130	42.098	42.539	42.145	
F6	FWA	163.692	137.155	137.203	137.736	137.487	137.547	137.299	
	EFWA	71.494	45.803	45.785	45.785	45.819	45.803	45.849	
	FWIS	88.198	60.253	60.811	60.166	61.281	60.745	59.544	
	FWIS	87.292	60.681	60.446	60.226	60.087	60.038	60.196	
	USATO	97.076	69.760	69.790	70.010	70.133	70.462	70.853	
SPSO	FWA	92.775	66.320	66.399	65.507	65.461	66.507	66.414	
	F7	FWA	126.905	126.252	126.934	126.733	126.844	126.702	126.889
	EFWA	34.801	34.179	34.753	34.177	34.192	34.238	35.691	
	FWIS	47.530	46.418	45.553	45.308	45.403	45.897	45.969	
	FWIS	47.006	44.536	44.396	44.256	44.162	43.991	44.037	
USATO	FWA	48.148	46.178	46.193	46.255	46.430	46.757	47.116	
	FWIS	46.567	46.412	46.349	45.911	46.834	46.569	47.177	
	F8	FWA	21.534	21.314	21.784	21.508	21.409	21.315	21.331
	EFWA	3.189	3.172	3.157	3.172	3.172	3.156	3.173	
	FWIS	5.031	4.909	4.875	4.892	4.891	4.969	4.892	
FWIS	FWA	6.825	6.829	6.769	6.734	6.595	6.689	6.656	
	USATO	8.360	8.267	8.267	8.283	8.359	8.298	8.298	
	SPSO	3.189	3.157	3.156	3.094	3.110	3.110	3.093	
	F9	FWA	26.269	25.550	25.550	25.534	25.533	25.535	25.534
	EFWA	8.079	7.485	7.220	7.313	7.361	7.376	7.407	
FWIS	FWA	19.517	18.082	17.617	17.298	17.313	17.278	17.315	
	FWIS	25.144	22.457	22.439	22.424	22.426	22.425	22.423	
	USATO	24.269	21.876	21.879	21.893	22.425	22.565	22.237	
	SPSO	3.094	3.079	3.047	3.063	3.031	3.095	3.031	
	F10	FWA	22.383	22.462	22.313	22.487	22.721	21.971	21.878
EFWA	FWA	3.735	3.642	3.665	3.619	3.641	3.641	3.638	
	FWIS	5.344	5.392	5.187	5.189	5.110	5.048	5.151	
	FWIS	6.313	6.235	6.251	6.189	6.265	6.345	6.344	
	USATO	7.032	7.079	7.142	7.110	7.219	7.079	7.250	
	SPSO	3.531	3.547	3.595	3.516	3.547	3.562	3.564	
F11	FWA	116.654	114.765	115.092	114.889	114.889	115.066	115.167	
	EFWA	24.316	23.861	24.191	24.512	24.412	25.424	24.692	
	FWIS	36.683	35.029	34.271	32.224	32.598	32.316	32.644	
	FWIS	36.239	33.833	33.880	33.850	33.895	34.036	34.365	
	USATO	39.787	36.989	37.036	37.177	37.176	37.301	37.271	
SPSO	FWA	34.957	35.629	35.770	35.941	35.926	36.255	36.333	
	F12	FWA	127.795	125.861	125.983	125.687	125.689	126.000	127.813
	EFWA	32.738	32.599	32.581	32.550	32.567	32.815	32.629	
	FWIS	39.990	39.770	40.004	40.144	39.614	39.566	39.583	
	FWIS	41.654	41.719	41.774	41.568	41.567	41.678	41.597	
USATO	FWA	44.848	43.895	43.864	43.770	43.646	43.833	44.115	
	FWIS	47.239	46.834	46.177	46.756	47.020	47.943	46.677	
	F13	FWA	120.702	119.719	119.702	119.828	119.810	119.733	119.748
	EFWA	28.441	27.753	27.754	27.784	27.833	27.956	28.081	
	FWIS	38.568	36.585	36.349	36.378	36.603	37.083	37.240	
FWIS	FWA	40.240	37.879	37.896	38.427	38.128	38.115	38.442	
	USATO	43.867	41.989	41.972	42.741	41.396	41.459	41.520	
	SPSO	45.052	44.256	43.911	43.944	44.412	45.005	44.913	

7.1. New Fitness Value Assigning Strategy

The new fitness value is represented as the strength of an individual combined with the density value of the individual, whereas the individual strength is the number of other individuals that the individual dominates. The fitness value is evaluated by the formula below:

$$f(x_i) = \sum_{(x_j \in P \cup NP) \wedge (x_j \succ x_i)} \cdot \left\{ x_j \in P \cup NP \mid x_i \succ x_j \right\} + \frac{1}{\sigma_k(x_i)} \quad (19)$$

The sign \succ represents the Pareto dominance relationship. $\sigma_k(x_i)$ is the distance of x_i to its k th nearest individual and k is set as the square root of the sample size $|P \cup NP|$.

7.2. New Mutation Strategy

MOFOA algorithm randomly selects three individuals and generates a new individual according to the following formula:

$$v_i = x_{r_1} + F \cdot (x_{r_2} - x_{r_3}) \quad (20)$$

In the formula, r_1 , r_2 and r_3 are random indexes of individuals. Parameter F is the scale factor.

The new individual will replace the old individual by a possibility of CR , as described in the following formula:

$$u_i^j = \begin{cases} v_i^j, & \text{if } rand(0,1) < CR \text{ or } j = i \\ x_i^j, & \text{otherwise} \end{cases} \quad (21)$$

Whenever a new individual is generated, it is compared with the old individual to find the better fitness value. For each generation, several individuals will be generated while the best fitness value will be selected.

The solutions of two algorithms are listed in Table 6 and the distribution is drawn as well in Figure 7.

Yet, FWA can work on 0/1 knapsack problems. The 0/1 knapsack problem is an NP hard problem and fireworks algorithm can solve this problem. Zhang J. Q. (2011) first used FWA on knapsack problem and obtained satisfactory solutions.

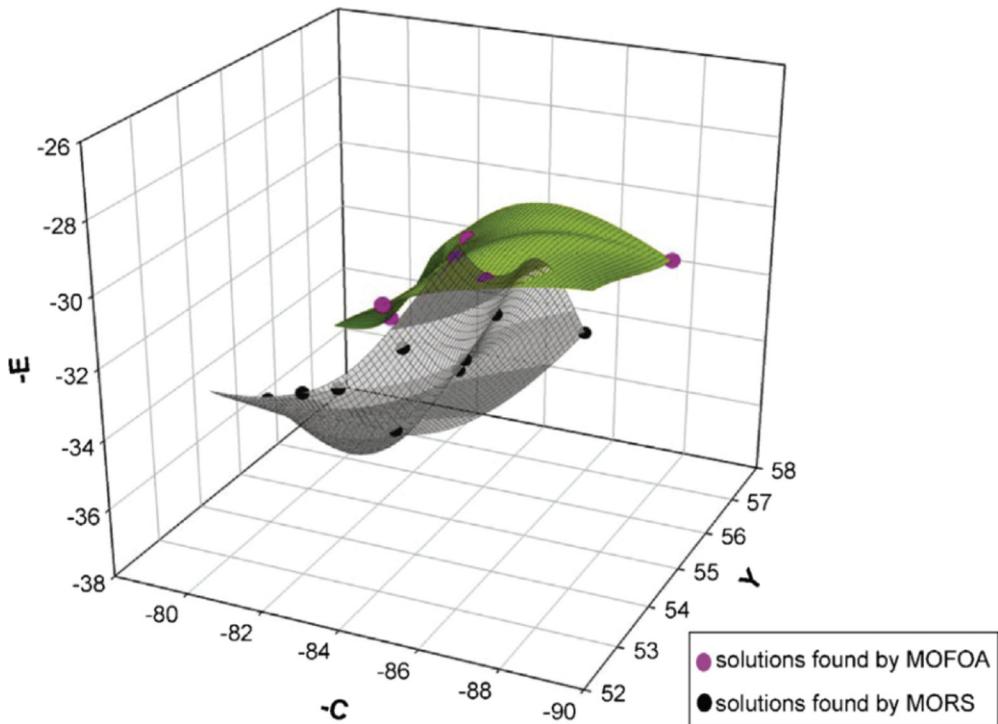
8. A GPU-BASED PARALLEL FIREWORKS ALGORITHM

Conventional swarm intelligence algorithms are not designed for GPU architecture. Therefore, they cannot make use of the tremendous computational ability of GPUs. As a result, they are difficult to deal with scaled problems. To overcome the shortcomings, many parallelized swarm intelligence algorithms are proposed

Table 6. Compared MOFOA with MORS on a variable-rate fertilization problem. This table is from the work of Zheng, Song, and Chen (2013).

Algorithms	Solution		
MORS	# 1(56.2, 86.5, 32.9)	# 2(54.8, 84.4, 33.2)	# 3(54.4, 85.7, 31.0)
	# 4(54.1, 85.2, 32.1)	# 5(53.8, 80.3, 34.0)	# 6(53.5, 84.0, 33.8)
	# 7(52.8, 84.9, 30.7)	# 8(52.6, 83.5, 31.9)	# 9(52.5, 82.7, 32.1)
MOFOA	# 1(57.3, 84.9, 31.4)	# 2(56.7, 82.9, 31.2)	# 3(56.4, 82.6, 31.5)
	# 4(56.1, 81.3, 33.4)	# 5(55.6, 83.6, 30.8)	# 6(54.8, 85.1, 30.5)

Figure 7. The distribution of solutions in the objective space for MOFOA and MORS. This figure is from the work of (Zheng, Song, & Chen, 2013).



to speedup conventional algorithms. Ding et al. (Ding, Zheng, & Tan, 2013) proposed an algorithm named GPU-based parallel fireworks algorithm (GPU-FWA).

GPU-FWA modifies conventional fireworks algorithm so that the algorithm is more suitable for GPU architecture. The implementation of GPU-FWA is based on the CUDA platform and the flowchart of GPU-FWA implementation on CUDA is given in Figure 8.

8.1. Two New Strategies

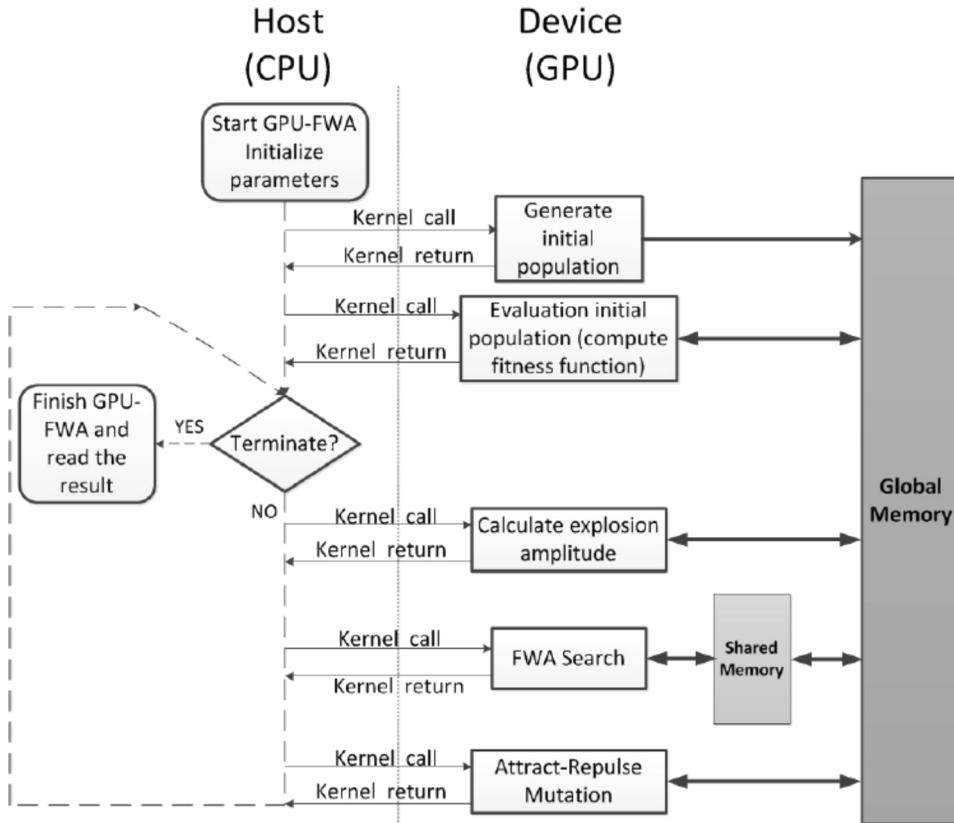
Two new strategies are introduced based on conventional FWA and GPU-FWA, namely FWA search and Attract Repulse Mutation. FWA search mimics the fireworks in the sky and generates a shower of sparks to explore the neighbor area. Attract repulse mutation is used

to keep the diversity of the fireworks swarm as it is vital to keep the diversity of a swarm in the optimization procedure. After the mutation operation, some sparks are close to the best spark, while some other sparks are distance from the best spark:

1. FWA Search (see Algorithm 2).

In FWA Search, each firework generates a fixed number of sparks independently. It takes a greedy strategy to decide which spark is selected as the new firework. It guarantees a strong capability of local search. In order to enhance the local search efficiency and minimize the overhead of communication, the greedy search is executed L times before the next strategy is triggered:

Figure 8. The flowchart of GPU-FWA implementation on CUDA. This figure is according with the work of (Ding, Zheng, & Tan, 2013).



Algorithm 2.

```

for i = 1 to L do
    generate m sparks.
    evaluate the fitness value of each sparks.
    find the current best spark with best fitness value.
    update the global best spark.
end for
    
```

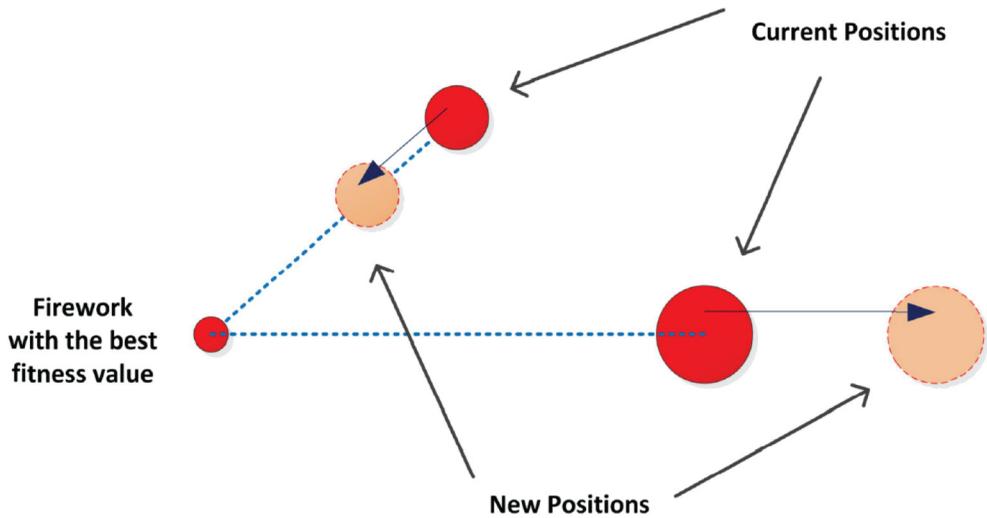
2. Attract Repulse Mutation.

In order to increase the diversity of fireworks, which is also to improve the global search capability, attract repulse mutation is applied after all fireworks have finished a round of FWA search. In attract repulse mutation, the firework with the best fitness value is selected as the center. Other fireworks can be either be

attracted to the center or repulsed away from it. This process is driven by picking a scaling factor randomly from a uniform distribution lying in $(1 - \delta, 1 + \delta)$ and the parameter δ varying from 0.1 to 0.9. After that, all dimensions are multiplied by the factor and fireworks are expected to fill the search space uniformly.

Figure 9 shows a general view of attract repulse mutation. The firework with the best

Figure 9. Attract repulse mutation. This figure comes from the work of Ding, Zheng, and Tan (2013).



fitness value is stationary (bottom left), while a position that is attracted will move towards it (top left) and another position that repels it moves away (bottom right), thus, creating two new positions.

The pseudo code of attract repulse mutation is shown in Algorithm 3.

Thanks to the two new strategies and the massively parallel computing hardware GPU,

GPU-FWA is much more effective than FWA and PSO. Compared with FWA and PSO which based on CPU, GPU-based GPU-FWA can reach a speedup as high as 250 times.

8.2. Implementation of GPU-FWA

Here are the three steps for the implementation of GPU-FWA:

Algorithm 3.

```

Initialize the new location:  $\widehat{x}_i = x_i$ ;
 $s = U(1 - \delta, 1 + \delta)$ ;
for d = 1 to D do
     $r = \text{rand}(0, 1)$ ;
    if  $r < \frac{1}{2}$  then
         $\widehat{x}_{i,d} = \widehat{x}_{i,d} + (\widehat{x}_{i,d} - x_{best,d}) \cdot s$ ;
    end if
    if  $\widehat{x}_{j,d} > ub_d$  or  $\widehat{x}_{j,d} < lb_d$  then
         $\widehat{x}_{j,d} = lb_d + |\widehat{x}_{j,d} - lb_d| \bmod (ub_d - lb_d)$ ;
    end if
end for

```

1. **Thread Assignment:** In the GPU-based parallel FWA, each firework is assigned to a group of threads (i.e. 32 continual threads). However, not all of the threads will necessarily be used in the computation. For instance, if the number of sparks is 16 and the number of a group of threads is 32, only half of the threads are used. By using thread assignment, three advantages are revealed. First of all, the threads in the same group can easily interchange information. Secondly, each group processes in the same space and the memory can be shared. As accessing the shared memory costs less time than accessing global memory, computational time can be greatly reduced. Thirdly, any proposed algorithm can be extended with problem scale since GPUs will automatically dispatch block while running;
2. **Data Organization:** For each firework, the position and fitness value are stored in the global memory. However, the data of sparks are stored in shared memory. Distinguished from interleaving configuration, both the data of fireworks and sparks are stored in a continuous way. This kind of organization is easy to extend with problem scale;
3. **Random Number Generation:** Since generating high quality random number is time

consuming, the efficient CURAND library (NVIDIA, 2012) is used for generating the random numbers in the implementation.

8.3. Experiments of GPU-FWA

Based on a state-of-the-art commodity Fermi GPU, extensive tests are taken on a suite of well-known benchmark function. GPU-FWA is compared with FWA and PSO on both running time and solution quality. Experimental results demonstrate that GPU-FWA generally outperforms both FWA and PSO, and enjoys a significant speedup as high as 200x, compared to the sequential version of FWA and PSO running on an up-to-date CPU. GPU-FWA also enjoys the advantages of being easy to implement and scalable.

Aside from running on CPU, conventional fireworks algorithm is also available to run on GPU. Comparing conventional FWA and GPU-based FWA, the experimental results are shown in the Table 7. Note that the functions below are different from the functions listed in the experiments design section.

For GPU-based parallel fireworks algorithm, the experimental results are tested on Windows 7 Professional x64 with 4G DDR3 Memory (1333 MHz) and Intel core i5-2310 (2.9 GHz, 3.1 GHz). The GPU used in the

Table 7. Comparison of GPU-FWA, FWA and PSO on mean and standard deviation

Functions No.	GPU-FWA		FWA		PSO	
	Average Value	Standard Deviation	Average Value	Standard Deviation	Average Value	Standard Deviation
F1	1.31e-09	1.85e-09	7.41e+00	1.98e+01	3.81e-08	7.42e-07
F2	1.49e-07	6.04e-07	9.91e+01	2.01e+02	3.52e-11	1.15e-10
F3	3.46e+00	6.75e+01	3.63e+02	7.98e+02	2.34e+04	1.84e+04
F4	1.92e+01	3.03e+00	4.01e+02	5.80e+02	1.31e+02	8.68e+02
F5	7.02e+00	1.36e+01	2.93e+01	2.92e+00	3.16e+02	1.11e+02
F6	-8.09e+03	2.89e+03	-1.03e+04	3.77e+03	-6.49e+03	9.96e+03
F7	1.33e+00	1.78e+01	7.29e-01	1.24e+00	1.10e+00	1.18e+00
F8	3.63e-02	7.06e-01	7.48e+00	7.12e+00	1.83e+00	1.26e+01

experiments is NVIDIA GeForce GTX 560 Ti with 384 CUDA cores, while the CUDA runtime version is 5.0. For more specific details of parameters setting, please see reference (Ding, Zheng, & Tan, 2013).

Both Table 7 and Table 8 are from previously published experiment results (Ding, Zheng, & Tan, 2013). The better results are shown in bold font. It can be seen from Table 7 and Table 8 that GPU-FWA defeated both FWA and PSO on the 8 functions, yet the GPU-FWA is also the quickest algorithm for calculation.

According to Table 8, it is obvious that GPU-FWA greatly reduced the running time compare with FWA and PSO.

For more details, please refer to the reference Ding et al (2013).

9. APPLICATIONS

Conventional fireworks algorithm and its variants are capable of dealing with optimization problems. Many researchers used these algorithms in a variety of applications.

Janecek et al. (2011) applied fireworks algorithm to non-negative matrix factorization (NMF). In their paper, a new iterative update strategy for multiplicative update algorithm based on fireworks algorithm is proposed. Experimental results have proved that the new iterative update strategy approach the same approximation error as the standard version in significantly fewer iterations. Besides, the new strategy consumes less time.

Gao et al. (2011) applied fireworks algorithm to digital filters design. After transforming the design of digital filters to a constrained optimization problem, fireworks algorithm was

able to find the global optimum. Computer simulations shown the filters using fireworks algorithm were better than using PSO and improved PSO algorithms.

He et al. (2013) used fireworks algorithm for spam detection. In their article, a new framework to optimize the anti-spam model using swarm intelligence optimization algorithm was proposed and experimental results show a good performance demonstrated on corpora PU1, PU2, PU3 and PUA.

Du (2013) solved nonlinear equations with fireworks algorithm and compared it with artificial bee colony (ABC) algorithm. From the four equations listed in his paper, fireworks algorithm was better than ABC algorithm on three equations. Therefore, fireworks algorithm worked very well on nonlinear equations.

9.1. FWA for NMF Computing

The Non-negative Matrix Factorization (NMF) refers to as low-rank approximation and has been utilized in several different areas such as content based retrieval and data mining applications, et al. NMF can reduce storage and runtime requirements, and also reduce redundancy and noise in the data representation while capturing the essential associations. The NMF method requires all entries in A , W and H to be zero or positive (Lee, & Seung 1999) which makes the interpretation of the NMF factors much easier. The NMF consists of reduced rank nonnegative factors $W \in \mathbb{R}^{m \times k}$ and $H \in \mathbb{R}^{k \times n}$ with $k \ll \min\{m, n\}$ that approximate a matrix $A \in \mathbb{R}^{m \times n}$ by WH . The nonlinear optimization problem underlying NMF can generally be stated as:

Table 8. Comparison of GPU-FWA, FWA and PSO on running time and speedup

Number of Sparks	FWA (s)	PSO (s)	GPU-FWA (s)	Speedup (FWA)	Speedup (PSO)
48	36.420	84.615	0.615	59.2	137.6
72	55.260	78.225	0.624	88.6	125.4
96	65.595	103.485	0.722	90.8	143.3
144	100.005	155.400	0.831	120.3	187.0

$$\begin{aligned} & \min_{W,H} f(W,H) \\ & = \min_{W,H} \frac{1}{2} \| A - WH \|_F^2 \end{aligned} \quad (22)$$

The error between the original data A and the approximation WH are stored in a distance matrix $D = A - WH$. The schematic diagram of coarse NMF approximation with extremely low rank k is shown in Figure 10.

To solve this problem, the nature-inspired optimization heuristics algorithms, genetic algorithms (Goldberg, 1988), particle swarm optimization (Kennedy, & Eberhart, 1995), differential evolution (Storn, & Price 1995), fish school search (Filho, de Lima Neto, Lins, Nascimento, & Lima, 2009), fireworks algorithm (Tan, & Zhu, 2010) are all used.

The parameters in the algorithms are set as following:

- **GA:** Mutation rate = 0.5; selection rate = 0.65;
- **PSO:** Following (Bratton, & Kennedy, 2007);
- **DE:** Crossover probability set to upper limit 1;
- **F S S :** $step_{ind_initial} = 1$, $step_{ind_final} = 0.001$, $W_{scale} = 10$;
- **FWA:** The number of first selected location is set as 10.

The experiment results of FWA on NMF computing are shown below. Figure 11 shows the convergence curves of accuracy while Figure 12 provides the running time for the six algorithms. It can be seen from the two figures that FWA works well on NMF computing.

Pseudo code for the iterative optimization for the Multiplicative Update (MU) algorithm

Figure 10. Scheme of coarse NMF approximation with extremely low rank k . This figure is firstly published in the work of Janecek, and Tan (2011).

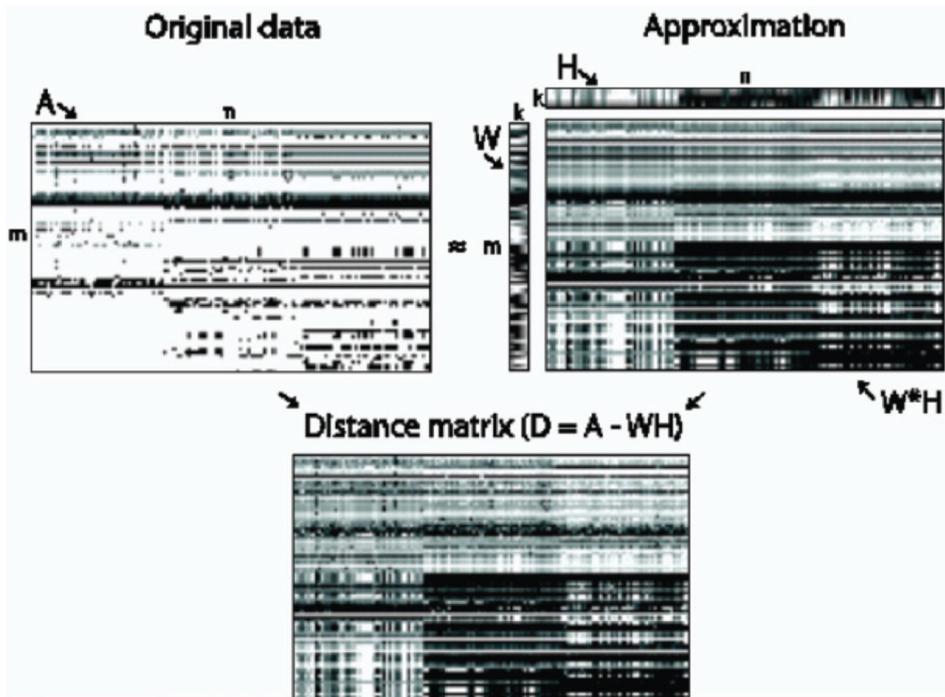


Figure 11. (a) Convergence curves of the accuracy when updating only the row of W , $m=2$, $c=20$, $k=2$ (b) Convergence curves of the accuracy when updating only the row of W , $m=2$, $c=20$, $k=5$. This figure is according to the work of Janecek, and Tan (2011).

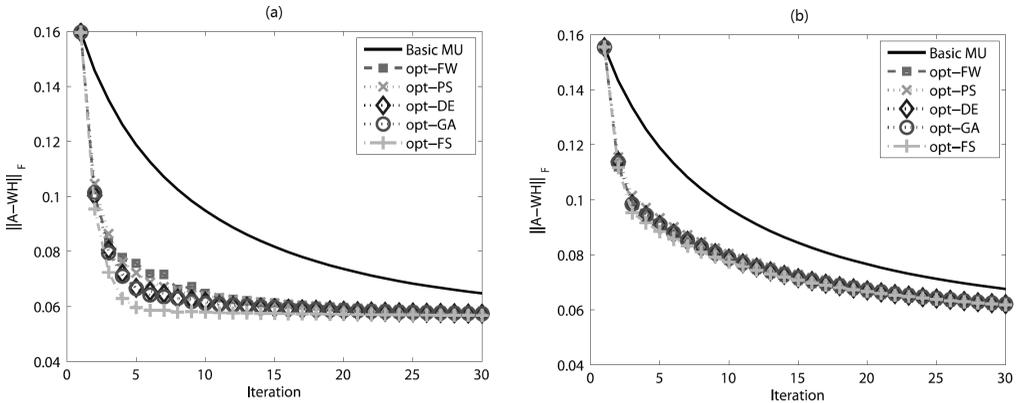
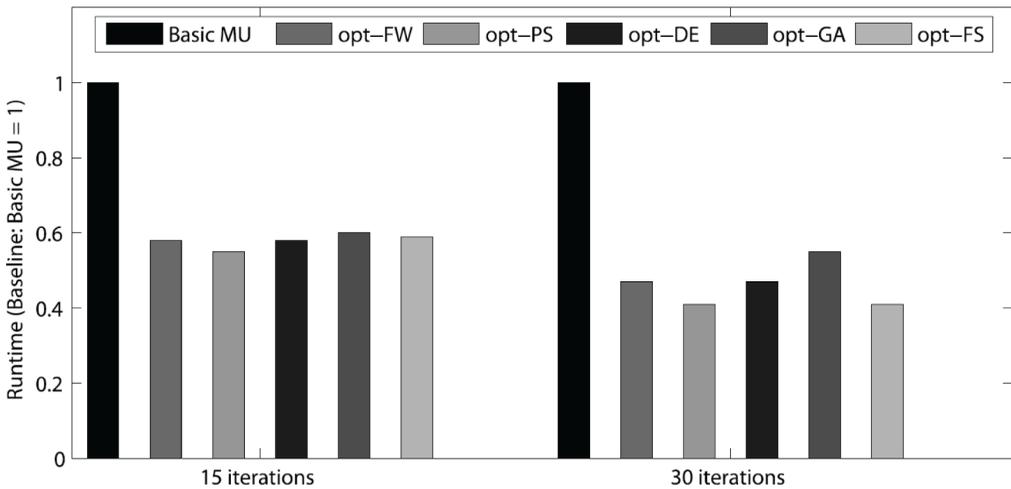


Figure 12. The proportion of running time to achieve the same accuracy. Set the running time of Basic MU as 1 and updating only the row of W , $m=2$, $c=20$, $k=2$. This figure comes from the work of Janecek, and Tan (2011).



is listed in Algorithm 4. The methods used in this algorithm are explained below. Here m is 2 while c is set as 20 which denotes the number of rows and/or columns that are optimized in the current iteration. Δc : The value of c is decreased by Δc in each iteration.

FWA performs just the same as the other heuristics algorithms while all of them get better results compared with Basic MU method,

the smaller the parameter k is, the more advantages the heuristics algorithms gain.

9.2. FWA on Design of Digital Filters

To design a digital filter, a multi-parameter optimization problem must be solved. However, the existing methods, such as particle swarm

Algorithm 4.

```

For T = 1 to maxIter dimensions do
    W = W · (AHT) / (WHHT + ε)
    H = H · (WTA) / (WTWH + ε)
    if t < m then
        dir is the ith row vector of D = A - WH
        [Val, IX_W] = sort(norm(dir), descend)
        IX_W = IX_W(1:c)
        ∀i ∈ IX_W, use SIO to find wir that minimized ||air - wirH0||
        W = [w1r; ...; wmr]
        dir is the ith row vector of D = A - WH
        [Val, IX_H] = sort(norm(dic), descend)
        IX_H = IX_H(1:c)
        ∀j ∈ IX_H, use SIO to find hjc that minimized ||ajc - Whjc||
        H = [h1c; ...; hnc]
    end if
end for

```

optimization (PSO), quantum-behaved particle swarm optimization (QPSO) and adaptive quantum-behaved particle swarm optimization (AQPSO) cannot find the optimal solution effectively. A cultural fireworks algorithm is proposed for digital filter design.

In the finite impulse response (FIR) and infinite impulse response (IIR) digital filters, cultural fireworks algorithm is used to design a joint objective function. The goal for cultural fireworks algorithm is to find the minimal value of the following function:

$$f(x) = \begin{cases} \alpha E_F + \beta E_I, & x \in s.t \\ \delta [\alpha E_F + \beta E_I], & x \notin s.t \end{cases} \quad (23)$$

$f(x)$ is the objective function while α and β are Boolean parameters. For FIR digital filters, α equals to 1 and β means to 0, while in IIR digital filters, verse vice. δ is set to be larger than 1, whereas E_F and E_I stands for

FIR and IIR filters separately. The constraint condition of vector x is represented as s.t. Table 9 shows the comparison of four algorithms in FIR filter design.

Figure 13 shows the flow chart of digital filter based on cultural fireworks algorithm (CFWA).

Experimental results show that the proposed cultural fireworks algorithm has a faster convergence and better optimization accuracy than particle swarm optimization, quantum-behaved particle swarm optimization and adaptive quantum-behaved particle swarm optimization algorithms. Therefore, cultural fireworks algorithm is effective and useful on digital filter design. For more experimental results, please refer to reference Gao, and Diao (2011).

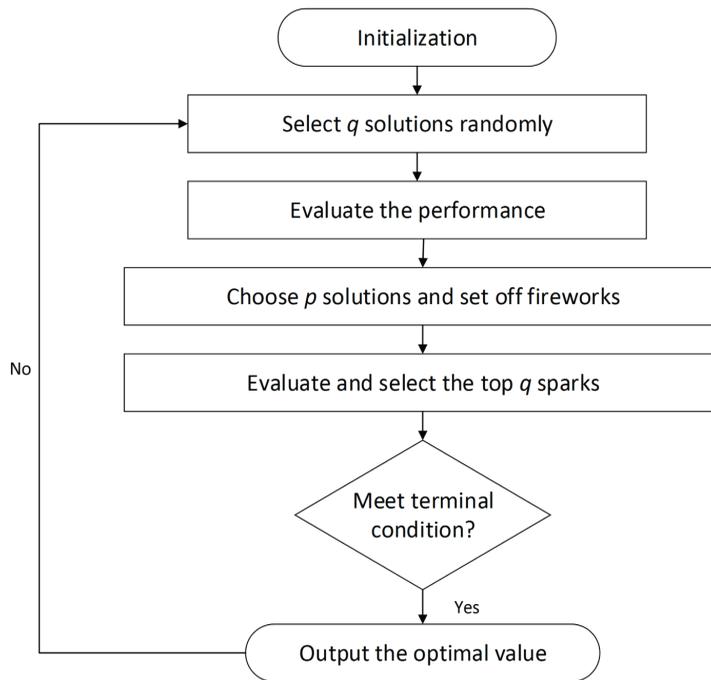
9.3. FWA on Spam Detection

In previous research, it is simple to set the parameters manually in the anti-spam process. However, the manually settings may cause

Table 9. Comparison of four algorithms on FIR filter. This figure is taken from the work of Gao, and Diao (2011).

Objective Value	Low-Pass Filter			
	PSO	QPSO	AQPSO	CFWA
Max	1.8505e-3	8.8845e-9	7.6096e-9	8.8818e-16
Min	7.5908e-5	6.6311e-12	1.1427e-10	0
Mean	3.9566e-4	5.8634e-10	1.2362e-9	2.5535e-16
Variance value	8.6053e-8	1.3445e-18	1.1087e-18	1.1330e-32

Figure 13. The flow chart of design digital filters by cultural fireworks algorithm. This figure is first published in the work of Gao, and Diao (2011).



several problems. First of all, when setting the parameters without prior knowledge, people have to test many groups of parameters to find the best one. Secondly, the parameters of different datasets are varied. There are no universal parameters.

To solve the problem of setting parameters, a new framework to find the proper parameters

in anti-spam model with fireworks algorithm is proposed. In the anti-spam model, the error rate represents the quality of the model. To make the error rate lower, an optimal vector:

$$P^* = \langle F_1^*, F_2^*, \dots, F_n^*, C_1^*, C_2^*, \dots, C_m^* \rangle$$

is suggested, which contains two parts. The first part F_1^* to F_n^* means the feature calculation relevant parameters and the second part C_1^* to C_m^* stands for the classifier relevant parameters. $CF(P)$ represents the cost function and it is calculated as:

$$CF(P) = Err(P) \tag{24}$$

where $Err(P)$ is the classification error of 10-fold cross-validation on the training set. Different feature extraction methods may need different parameters. In local-concentration model, the selection rate m helps to select the top m percent terms in a term set with descending importance. The proclivity threshold θ equals to the minimal difference of a term's frequency in non-spam emails minus a term's frequency in spam emails. The parameter N is the number of sliding windows.

The flowchart of using fireworks algorithm to optimize parameters in local-concentration model for spam detection is given in Figure 14.

Fireworks algorithm is used to optimize the parameters in the model and there are two strategies to build the model. The first strategy is to derive a small dataset from the training set. The small dataset is used as a validation set and do not participate in the training process. After building a model on the training set, the model is validated on the small dataset. The best model is chosen before apply to the test set. The second strategy is to divide the training set into ten even parts and each part is used only once as the validation set. Therefore, ten models are built and the best model is applied to the test set.

Table 10 shows the experimental results of the comparison between fireworks algorithm with two strategies each with local concentration (LC) method.

Tables 10 and 11 are from the work of He, Mi, and Tan (2013). The details of evaluation criteria can be found in He, Mi, and Tan (2013).

Experimental results show that the fireworks algorithm is better than local concentrate method on corpora PU1, PU2, PU3 and PUA.

9.4. FWA on Non-Linear Equations

In the engineering and scientific fields, many problems can be transferred to non-linear equations. Traditional methods use derivative of the object function to solve non-linear equations. However, traditional methods are sensitive to initial values and convergent in local area. Thus, swarm intelligent algorithms are used to solve non-linear equations. Since artificial bee colony algorithm cannot achieve the best optimal result, fireworks algorithm is used to deal with non-linear equations.

Four non-linear equations from article Du (2013) are listed below:

Equation 1:

$$f(x) = x^3 - 2x - 5 = 0, x \in [-4, 4]$$

Equation 2:

$$f(x) = x^3 - 3x^2 - 6x + 8, x \in [0, 2]$$

Equation 3:

$$\left\{ \begin{array}{l} f_1(x) = x_1 + \frac{1}{4}x_2x_4x_6 + 0.75 = 0 \\ f_2(x) = x_2 + 0.405e^{(1+x_2)} - 1.405 = 0 \\ f_3(x) = x_3 - \frac{1}{4}x_4x_6 + 1.25 = 0 \\ f_4(x) = x_4 - 0.605e^{(1-x_3^2)} - 0.395 = 0 \\ f_5(x) = x_5 - \frac{1}{2}x_2x_6 + 1.5 = 0 \\ f_6(x) = x_6 - x_1x_5 + 1.5 = 0 \end{array} \right.$$

where x_i varies from -2 to 2 and the optimal lies in (-1, 1, -1, 1, -1, 1)T:

Figure 14. The flowchart of parameter optimization in a local-concentration model for spam detection using fireworks algorithm. This figure is first published in the work of (He, Mi, & Tan, 2013).

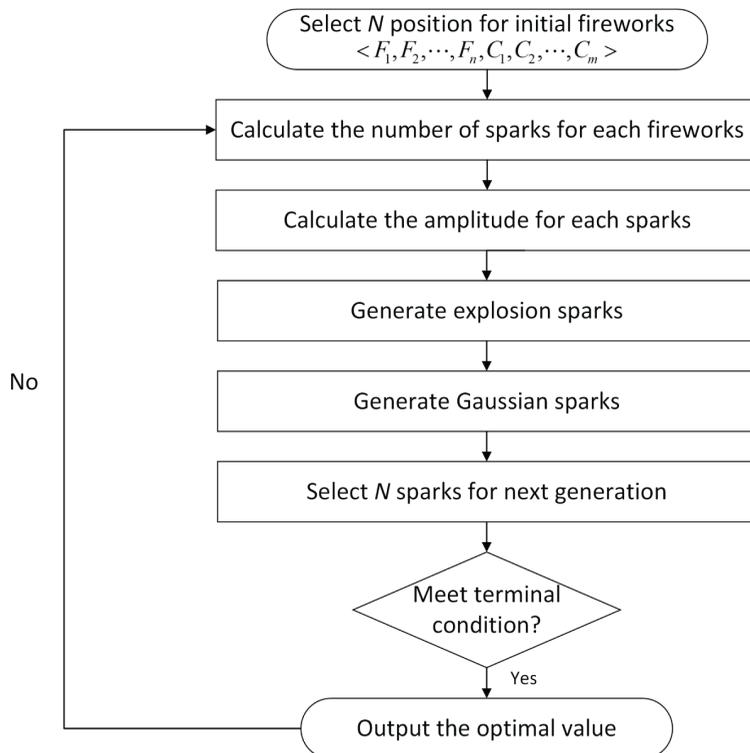


Table 10. Comparison of fireworks algorithm with the first strategy and local concentration method

Corpus	Methods	Precision (%)	Recall (%)	Accuracy (%)	F ₁ -Measure
PU1	LC	94.85	95.63	95.87	95.21
	FWA	96.55	95.21	96.33	95.81
PU2	LC	95.74	77.86	94.79	85.16
	FWA	95.15	80.71	95.35	86.65
PU3	LC	96.68	94.34	96.03	95.45
	FWA	95.81	95.71	96.18	95.69
PU4	LC	95.60	94.56	94.91	94.94
	FWA	96.63	94.56	95.53	95.49

Table 11. Comparison of fireworks algorithm with the second strategy and local concentration method

Corpus	Methods	Precision (%)	Recall (%)	Accuracy (%)	F ₁ -Measure
PU1	LC	100.00	92.36	96.67	95.88
	FWA	100.00	96.64	98.57	98.22
PU2	LC	100.00	64.00	90.71	74.62
	FWA	100.00	94.17	98.57	96.57
PU3	LC	97.84	91.30	95.37	94.34
	FWA	98.25	95.91	97.56	97.02
PU4	LC	95.78	90.72	93.64	92.68
	FWA	98.75	96.44	97.73	97.42

Equation 4:

$$\begin{cases} x_1^{x_2} + x_2^{x_1} - 5x_1x_2x_3 - 85 = 0 \\ x_1^3 - x_2^3 - x_3^3 - 60 = 0 \\ x_1^{x_3} + x_3^{x_1} - x_2 - 2 = 0 \end{cases}$$

where the ranges of x_i is from 0 to 10 and the best solution is (4, 3, 1)T.

Note that the square of each equation is the objective function. The steps of fireworks algorithm is as follows:

Step 1: Randomly generates n individuals at initial;

Step 2: Generate common sparks and Gaussian sparks the same as fireworks algorithm;

Step 3: Choose the best individual for next generation and the next $(N - 1)$ individuals are choose the same like fireworks algorithm;

Step 4: If the terminal condition is met, stop the procedure. If not, go back to step 2.

The experimental result is listed in Table 12. The best results are in bold font.

Table 12. Comparison of ABC and FWA on four equations. The table comes from the work of Du (2013).

Equation No.	Variables	ABC	FWA
Equation 1	X	2.09455448	2.09465148
Equation 2	X	1.14324234	0.98973242
Equation 3	X1	-1.000343	-1
Equation 3	X2	0.844234	1
Equation 3	X3	-1.64535	-1
Equation 3	X4	1.031231	1
Equation 3	X5	-0.98232	-1
Equation 3	X6	0.9932432	1
Equation 4	X1	4.000013	4
Equation 4	X2	3.000029	3
Equation 4	X3	0.962344	1

The result of artificial bee colony (ABC) algorithm is from the work of Zhang J. L. (2012).

10. CONCLUSION AND FUTURE DIRECTIONS

Fireworks algorithm provides a brand new way to solve complex problems. The current fireworks algorithm and its applications prove that it can solve many optimization problems effectively. Furthermore, fireworks algorithm can be parallelized and thus suitable to deal with big data problems. No matter for theoretical or applied researches, fireworks algorithm is worth researching and can bring great scientific and economic benefits.

However, there are still some disadvantages in fireworks algorithm. Firstly, fireworks algorithm simulates behaviors of biomes and lack of necessary mathematical foundation. For example, there is no proof of convergence in fireworks algorithm. Secondly, most of the parameters in fireworks algorithm are set by experience and the parameters largely depend on specific problems. Thirdly, not many applications of fireworks algorithm are currently in use. Furthermore, it is crucial to observe each algorithm in real world problems, rather than strictly theoretical situations in order to fully appreciate its benefits.

Fireworks algorithm has been greatly developed, but still it is not perfect. The direction of its future development can be described as follows. First of all, fireworks algorithm needs its mathematical foundation and theoretical analysis. Secondly, the selection of the control parameters of fireworks algorithm often relies on experience. So how to choose the mostly appropriate parameters needs a theoretical guidance. Thirdly, the prospects of fireworks algorithm applications are still at infancy and require further exploration. Fourthly, as an open source algorithm, fireworks algorithm can learn from other algorithms. How to improve fireworks algorithm is also a useful research

direction. Last but certainly not least, the study of GPU to accelerate fireworks algorithm is at its initial stage and will attract more and more researchers who are devoted to apply the FWA to real world problems.

ACKNOWLEDGMENT

This work was supported by the Natural Science Foundation of China with grant no. 61375119, 61170057 and 60875080.

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