Enhancing Fireworks Algorithm in Local Adaptation and Global Collaboration for Solving ICSI 2022 Benchmark Problems

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Abstract. Fireworks algorithm is a swarm intelligence optimization algorithm with superior performance, which can be used to solve various practical optimization problems. To enhance the performance of fireworks algorithm, we introduce a powerful local search mechanism and add multiple cooperative strategies. These strategies improve the local exploitation capability and global exploration capability of fireworks algorithm. The experimental results on the ICSI'2022 test set demonstrate that the performance of the algorithm is satisfactory.

Keywords: Fireworks algorithm · Swarm intelligence · Optimization · Collaboration optimization

1 Introduction

Optimization problem has always been one of the hottest topics of research in various fields because it is widely used in many real-world applications, especially with the advent of machine learning and deep learning. Due to the complexity of modern optimization problems, the optimization of some functions is relatively difficult. The emergence of stochastic search algorithms such as swarm intelligence optimization algorithms and evolutionary algorithms makes it possible to find the global optimal solutions of some complex functions.

Fireworks algorithm (FWA [7]) is a kind of swarm intelligent optimization algorithm inspired by the phenomenon of firework explosion. During the process of fireworks algorithm, fireworks create sparks around themselves by exploding, which could search the surrounding area. Besides, fireworks could cooperate with each other to improve the efficiency of search. After multiple iterations, the algorithm is likely to find the global optimum of the objective function.

The single-objective bounded optimization problem is one of the basic settings of all optimization problems. Many complex optimization problems can be decomposed into single-objective optimization problems. Therefore, for many current swarm intelligence optimization algorithms and evolutionary algorithms,

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how to improve the performance of the algorithm on single-objective optimization problems is a key issue to deal with.

In this paper, we introduce a novel fireworks algorithm. It introduce the local search mechanism in CMA-ES to improve the local search performance of the fireworks algorithm. In addition, a new search space partition strategy has been added to the algorithm to improve the collaborative ability of fireworks, which greatly enhances the global search ability of the fireworks algorithm. The newly proposed algorithm is called Fireworks Algorithm with Search Space Partition (FWASSP). The ICSI'2022 test set is a newly proposed single-objective optimization test set for various intelligent optimization algorithms. We carry out experiment on ICSI'2022 with FWASSP. The experimental results show that the new algorithm performs well in both global search and local search.

The paper is organized as follows. Section 2 shows the background of our research, including the problem definition and related works. Section 3 describes the newly proposed algorithm in detail. The experimental results on the ICSI'2022 test set and the discussion are shown in Sect. 4.

2 Background

2.1 Problem Definition

Without loss of generality, we consider the general bound-constrained optimization problem which targets to find the optimal solution x^*:

$$\mathbf{x}^* = \arg\min_{x \in S} f(\mathbf{x}), \tag{1}$$

where $f : \mathbb{R}^d \to \mathbb{R}$ is an unknown objective function (also called fitness function). $S = \left\{x \in \mathbb{R}^d : lb_i < x_i < ub_i\right\}$ is the feasible space of f. lb_i is the lower bound of x_i and ub_i is the upper bound of x_i.

2.2 Related Works

Evolution Strategies (ESs) are a sub-class of nature-inspired optimization methods belonging to the class of Evolutionary Algorithms (EAs). Covariance Matrix Adaptation Evolution Strategy (CMA-ES) [1] is a well-designed evolutionary strategy. It uses the quadratic model to fit local shapes to improve search efficiency. Besides, quite a few mechanisms are employed to control the search direction and the step size. Therefore, CMA-ES has an excellent ability in local search.

Tan and Zhu proposed a firework algorithm by simulating the explosion of fireworks [7]. After fireworks algorithm was proposed, it received extensive attention due to its great performance and excellent optimization efficiency. On the basis of the fireworks algorithm, researchers have proposed many variants of the fireworks algorithm such as EFWA [9], AFWA [3], dynFWA [8], CoFFWA [10] and GFWA [4]. In 2017, Li and Tan proposed the Loser-out Tournament-based Fireworks Algorithm (LoTFWA) [2], which added the loser elimination mechanism to reinitialize the noncompetitive fireworks. This mechanism could restart the fireworks which are trapped in the local optimum to accelerate the process of optimization.

In LoTFWA, each firework optimizes its local area by a uniform explosion within a dynamic amplitude. A guided mutation spark is generated for each firework to accelerate its local exploitation. Then, some unpromising fireworks are detected and restarted to avoid waste of resources. Algorithm 1 outlines the framework of LoTFWA. A detailed explanation and parameter setting of LoTFWA can be found in [2].

Algorithm 1. Loser-out Tournament-based Fireworks Algorithm

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1: Randomly initialize μ fireworks in the search space.
2: Evaluate the fireworks' fitness
3: repeat
4:     for i = 1 to μ do
5:         Calculate dynamic explodes parameters λ_i and A_i.
6:         Generate explosion sparks.
7:         Generate guiding sparks.
8:         Evaluate all the fitness of the sparks.
9:         Select the best individual (including firework, its explosion sparks and guiding sparks) as the next generation of fireworks.
10:    Perform the loser-out tournament.
11: until Termination criterion is met.
12: return The position and the fitness of the best solution.
```

3 Proposed Strategies

LoTFWA is an outstanding global optimization algorithm with extremely simple mechanisms. However, there are still two major weaknesses in LoTFWA which need to be improved.

1. The local search efficiency of the explosion operator and mutation operator is limited by a basic uniform trust region scheme. This results in the searched solution being less refined.
2. The collaboration method is too weak because the restart mechanism is rarely triggered and it can only save limited resources rather than guide fireworks to cooperate.

In response to the above problems, we propose a series of strategies to improve it.

3.1 Gaussian Explosion with Adaptation

For the first weakness, the local search capability of the fireworks algorithm is enhanced by introducing the local search strategy in the CMA-ES [1]. CMA-ES uses Gaussian explosion instead of uniform explosion. The advantage of Gaussian explosion is that it has more parameters to control the shape of the explosion, which means that the search efficiency will be higher. By introducing Gaussian explosion, FWASSP is able to estimate the local fitness landscape and generate more effective sparks.

In the g-th generation, the k-th explosion spark $\mathbf{x}_k^{(g+1)}$ is generated from a Gaussian distribution:

$$\mathbf{x}_k^{(g+1)} \sim \mathbf{m}^{(g)} + \sigma^{(g)} \times \mathcal{N}\left(\mathbf{0}, C^{(g)}\right), \tag{2}$$

where $\mathbf{m}$ and C is the mean and covariance matrix. $\sigma^{(g)}$ is the overall step size. In the g-th generation generation, each firework generates the same number of λ sparks.

After evaluation of all sparks $\mathbf{x}^{(g+1)}$, the explosion distribution is adapted according to the strategies in CMA-ES. The detailed explanation and parameter setting of CMA-ES can be found in [1]. The complete adaptation process is provided in [5].

3.2 Restart Mechanism

Since Gaussian explosion and adaptation mechanism accelerate local optimization process significantly, the algorithm requires more detection mechanisms to ensure timely restart of fireworks that are not promising to improve the global optimal. Four extra restart conditions are proposed in our algorithm. These condition are determined by the search status of the firework individual and the relationship between fireworks:

1. Low Value Variance: $\mathrm{var}\left[f\left(x_{1:\lambda}^{(g+1)}\right)\right] \leq \epsilon_v$
2. Low Position Variance: $\sigma^{(g+1)} \times \left\|\mathbf{C}^{(g+1)}\right\| \leq \epsilon_p$
3. Not improving: Not improved for $I_{max\_not\_improve}$ iterations.
4. Covered by Better: More than 85% of the firework's sparks are covered by a better firework's explosion range.

3.3 Collaboration

Since we use Gaussian explosion, the explosion boundary of a firework X with parameters $(\mathbf{m}, C, \sigma)$ is defined as:

$$\left\{\mathbf{x} \| \|\mathbf{C}^{-\frac{1}{2}}\left(\frac{\mathbf{x}-\mathbf{m}}{\sigma}\right)\| = E\|\mathcal{N}(\mathbf{0}, \mathbf{I})\|\right\}. \tag{3}$$

There are two principles in collaboration strategies. First, the explosion scope tends to form a segmentation within the global optimization area, which can help fireworks avoid overlapping or omission of search scope. Second, the better fireworks tend to search independently, and the worse fireworks tend to search collaboratively. It guarantees the local optimization of leading fireworks will not be severely affected by collaboration.

Based on these ideas, the proposed algorithm conducts collaboration by the following steps:

Compare Fireworks. We need to compare the search progress of the fireworks for collaboration strategies. A fuzzy comparison between each pair of fireworks is introduced to estimate their relative optimization progress, which is described in Algorithm 2.

Algorithm 2. Fuzzy Comparison of Fireworks

Require: Fireworks $\mathbf{X}_i$ and $\mathbf{X}_j$ with sparks $\mathbf{x}_{i,1:\lambda}^{(g+1)}$ and $\mathbf{x}_{j,1:\lambda}^{(g+1)}$ (if not restarted)
1: **if** Both $\mathbf{X}_i$ and $\mathbf{X}_j$ are just restarted **then**
2: **return** $\mathbf{X}_i$ and $\mathbf{X}_j$ are similar
3: **if** $\mathbf{X}_i$ is restarted **then**
4: **return** $\mathbf{X}_j$ is ahead of $\mathbf{X}_i$
5: **else**
6: **return** $\mathbf{X}_i$ is ahead of $\mathbf{X}_j$
7: **if** $\min \mathbf{x}_{i,1:\lambda}^{(g+1)} > \max \mathbf{x}_{j,1:\lambda}^{(g+1)}$ **then**
8: **return** $\mathbf{X}_j$ is ahead of $\mathbf{X}_i$
9: **else**
10: **return** $\mathbf{X}_i$ is ahead of $\mathbf{X}_j$
11: **return** X_i and X_j are similar

The fuzzy comparison method saves the time of the algorithm. At the same time, it can provide enough accurate information.

Compute Dividing Points. Different fireworks cooperate to search different areas, so it is necessary to calculate the dividing points to specify where the search range of both fireworks are divided. Figure 1 shows 4 possible situations of the collaboration method. We use the following steps to calculate the dividing point, which is described in Algorithm 3.

Fit Dividing Points. The boundary of firework $X(\mathbf{m}, C, \sigma)$ is adapted to fit its dividing points. For each dividing point P_k, a new covariance matrix C_k is calculated. On the direction of XP_k, P_k lies right on the boundary. On the conjugate directions, the radii of boundary do not changed. The mean of all adapted covariance matrix $\frac{1}{K}\sum_{k=1}^{K} C_k$ is taken as the overall collaborated result of X.

The mathematical calculation for fitting a single split point can be found in [5].

Algorithm 4 outlines the framework of the proposed collaboration strategy:

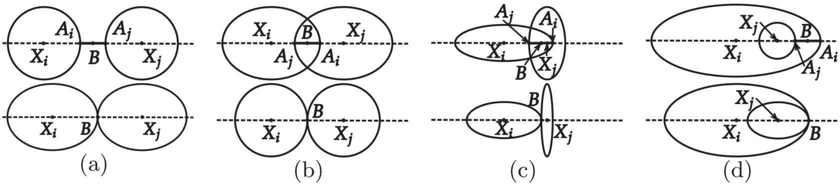

Fig. 1. Four cases of collaboration between two fireworks. A_i and A_j are the closer intersections of line X_iX_j with their boundaries. The actual dividing point could be any point on A_iA_j. The second row shows the collaboration results when taking the midpoint B of A_iA_j as dividing point.

Algorithm 3. Compute Dividing Points

Require: Fireworks $\mathbf{X}_i$, $\mathbf{X}_j$ and their intersections A_i, A_j

Calculate the radius $r_{ij} = \|X_iA_i\|$ and $r_{ji} = \|X_jA_j\|$ on line X_iX_j
Determine the situation (See Fig. 1) according to r_{ij}, r_{ji} and d_{ij}
Calculate the position of A_i and A_j
if X_i is ahead of X_j **then**
    A_i is the dividing point
else if X_j is ahead of X_i **then**
    A_j is the dividing point
else
    the midpoint B of A_iA_j is the dividing point.
return X_i and X_j are similar

Algorithm 4. Framework of Fireworks Collaboration

Require: n fireworks X_i and parameters $(\mathbf{m}_i, \mathbf{C}_i, \sigma_i)$ in N dimensional feasible space
Ensure: Collaborated parameters of fireworks

for each pair of fireworks X_i and X_j **do**
    Compare the progress of X_i and X_j
    Calculate $d_{ij} = |X_iX_j|$, expected sample distance r_{ij} and r_{ji} on X_iX_j
    Calculate the dividing point P_{ij} $(= P_{ji})$
for each firework X_i **do**
    Gather $K = \min(N, n-1)$ closest dividing points $P_{i,j_{1:K}}$
    Clip the length of $X_iP_{ij_k}$ within $[0.5r_{ij_k}, 2r_{ij_k}]$
    for $k \leftarrow 1 : K$ **do**
        Fit P_{ij_k} on the boundary of X_i and obtain $\mathbf{C}_{ij_k}$
    $\mathbf{C}_i \leftarrow \frac{1}{K}\sum_{k=1}^{K}\mathbf{C}_{ij_k}$

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4 Experimental Results

The performance of proposed algorithm is tested on objective functions from the ICSI 2022 benchmark test set. This test set contains 10 black-box test functions, including 3 unimodal functions, 5 multimodal functions and 3 composition functions.

According to the settings of the single-objective optimization competition, each function is tested for 50 repetitions with 10, 20, 50 dimensions. The termination condition is a maximum of 10,000, 30,000 or 70,000 evaluations for 10, 20 or 50 dimensions respectively.

To demonstrate the efficiency of our proposed algorithm, the proposed algorithm is compared with three baselines. LoTFWA [2] is the most efficient one of the main variants of the firework algorithm. CMA-ES [1] is an excellent evolutionary algorithm with outstanding local optimization ability. APGSK-IMODE [6] is a variant of differential evolution algorithm. It has achieved excellent results in the CEC 2021 competition. All these algorithms are tested under the same conditions as the proposed algorithm.

The parameters of all the tested algorithms are set as follows. Its basic settings and parameters are as same as LoTFWA, which includes 5 fireworks and each firework generates 300 sparks in each iteration. In the restart conditions, ϵ_v and ϵ_p are both $1E-12$, and the maximum number of unimproved iteration $I_{max\_unimprove}$ is 150. The parameters of local adaption is also set to be the same as CMA-ES. As we can see, our algorithm does not choose different parameters according to the specific problem. In order to ensure a fair comparison, the parameters of other algorithms are set according to the settings in their original papers.

The statistical results of the four algorithms are shown in the Table 1, Table 2 and Table 3 for 10 D, 20 D and 50 D respectively. Each table contains the mean errors and the mean standard deviations of four algorithms on ICSI'2022 test set. In addition, these algorithms are ranked according to their mean errors on each function, and the average rankings (ARs) over the 10 functions are presented at the bottom of the table. Their statistical information is shown in Fig. 2.

It can be seen from the experimental results that the performance of the algorithm are excellent, whether it is on unimodal or multimodal functions. The performance of the algorithm on the composition function is slightly worse, because the composition function is more complicated. On high-dimensional problems, it has also achieved very good optimization results compared with other baseline algorithms. This is because the global collaboration strategy of the algorithm can make the algorithm avoid trapping in many local optimal values.

On unimodal functions, both CMA-ES and proposed algorithm performs well because they have strong local exploitation mechanisms. On multimodal functions, our proposed algorithm performs the best. The reason for the excellent performance of our newly proposed algorithm is the algorithm is composed of multiple populations. The collaboration mechanism of them saves search resources thus it can find better global optimum in complex multimodal functions. As for composition functions, due to the complexity of the function, the performance of the newly proposed algorithm is comparable to CMA-ES.

Table 1. Results for 10 D problems

F.	LoTFWA		APGSK-IMODE		CMA-ES		Proposed	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std
1	7.02e+04	7.18e+04	1.24e−01	5.87e−01	1.89e+00	9.99e+00	4.34e−03	7.77e−03
2	9.02e+02	1.50e+03	8.14e−03	2.53e−02	8.87e−14	2.30e−13	3.23e−08	5.94e−08
3	1.45e+00	2.23e+00	6.30e+00	0.00e+00	6.30e+00	1.08e−06	6.30e+00	3.46e−07
4	9.24e+00	2.93e+00	4.38e+00	1.31e+00	8.76e−01	7.87e−01	4.78e−01	5.35e−01
5	3.15e+02	1.20e+02	2.50e+02	1.11e+02	1.96e+01	5.78e+01	1.50e+01	3.57e+01
6	2.23e−01	3.47e−01	2.24e−01	4.49e−01	9.64e−04	2.07e−03	4.58e−09	8.40e−09
7	1.18e−01	4.29e−02	8.40e−02	3.57e−02	1.22e−01	3.91e−02	1.97e−02	5.06e−03
8	2.28e+05	2.04e+05	6.51e+04	1.45e+05	2.99e+05	1.69e+05	2.65e+05	1.90e+05
9	1.59e+03	2.84e+03	2.19e+04	1.67e+04	7.03e−02	1.31e−01	1.01e−03	7.03e−03
10	1.62e+01	9.20e+00	8.38e+00	6.58e+00	2.05e+01	1.23e+01	4.04e−01	2.20e−01
AR.	3.10		2.60		2.80		1.50	

Table 2. Results for 20 D problems

F.	LoTFWA		APGSK-IMODE		CMA-ES		Proposed	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std
1	3.64e+05	1.82e+05	9.82e+04	7.05e+04	6.95e−11	1.80e−10	5.56e−08	1.06e−07
2	1.14e+02	2.34e+02	1.45e+03	6.87e+02	0.00e+00	0.00e+00	9.50e−13	4.11e−13
3	3.62e+00	4.51e+00	9.77e+00	3.69e−06	9.77e+00	1.00e−07	9.77e+00	1.37e−07
4	2.80e+01	6.14e+00	2.31e+01	3.85e+00	2.19e+00	1.50e+00	8.56e−01	7.71e−01
5	1.26e+03	2.17e+02	1.30e+03	2.47e+02	1.78e+02	1.26e+02	9.21e+01	6.62e+01
6	1.80e+00	1.64e+00	2.68e+01	5.70e+00	1.14e−13	0.00e+00	7.22e−11	2.93e−11
7	2.74e−01	5.98e−02	1.83e−01	2.58e−02	1.33e−01	3.72e−02	2.05e−02	5.52e−03
8	4.29e+05	5.93e+03	4.48e+05	5.91e+03	4.03e+05	4.58e+02	4.03e+05	3.67e+02
9	2.21e+04	4.45e+04	1.30e+07	3.91e+06	1.05e+01	6.17e+01	2.43e+06	9.77e+06
10	4.99e+01	2.76e+01	1.11e+01	4.56e+00	1.92e+01	9.84e+00	6.55e−01	3.05e−01
AR.	2.90		3.00		1.80		1.40	

Table 3. Results for 50 D problems

F.	LoTFWA		APGSK-IMODE		CMA-ES		Proposed	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std
1	5.60e+06	1.34e+06	1.62e+07	2.86e+06	0.00e+00	0.00e+00	4.23e−12	9.60e−13
2	8.30e+01	1.31e+02	6.69e+01	5.97e+01	0.00e+00	0.00e+00	1.83e−12	1.59e−13
3	2.71e+01	3.16e+01	2.01e+00	1.02e+01	0.00e+00	0.00e+00	1.93e−12	1.30e−13
4	1.21e+02	1.80e+01	1.84e+02	1.78e+01	6.09e+00	2.74e+00	1.75e+00	9.02e−01
5	4.98e+03	4.55e+02	6.32e+03	2.85e+02	1.15e+03	3.89e+02	3.65e+02	9.69e+01
6	9.61e+00	3.92e+00	3.71e+02	3.94e+01	4.41e−13	2.91e−14	2.11e−10	3.55e−11
7	4.81e−01	7.57e−02	3.39e−01	2.57e−02	1.74e−01	3.48e−02	3.75e−02	7.28e−03
8	4.57e+05	8.85e+03	5.73e+05	9.10e+03	4.07e+05	6.99e+02	4.06e+05	2.62e+02
9	9.62e+05	1.19e+06	2.07e+09	3.77e+08	3.04e−03	5.67e−03	1.73e−09	3.74e−10
10	7.61e+02	2.07e+02	3.62e+02	3.57e+01	3.24e+02	4.97e+01	1.62e+02	8.21e+00
AR.	3.00		3.20		1.60		1.00	

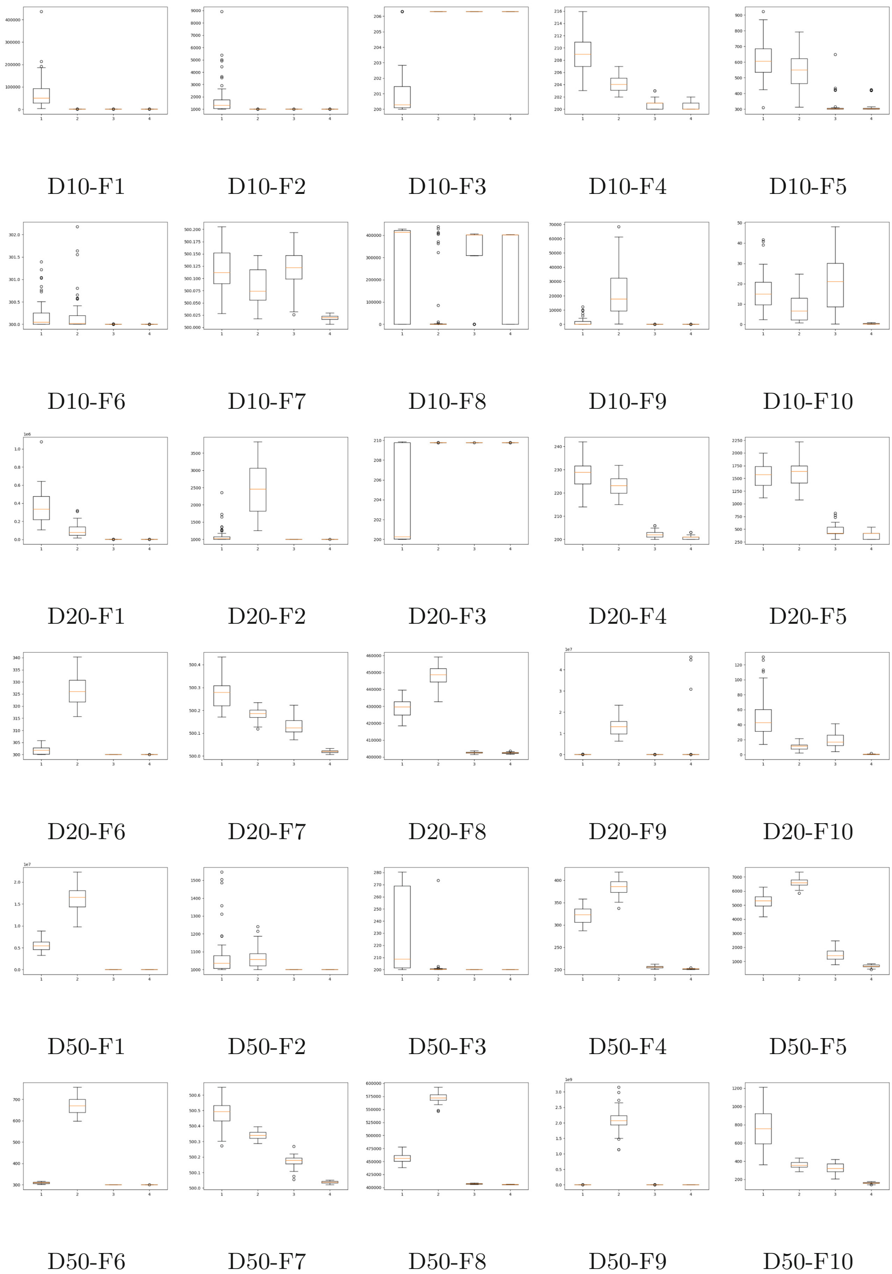

Fig. 2. Boxplots of the four algorithms

In conclusion, the newly proposed algorithm has a significant improvement over LoTFWA. Compared with other classic heuristic algorithms, FWASSP also has an extremely good performance. It is worth further investigation.

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References

1. Hansen, N.: The cma evolution strategy: a comparing review. In: Lozano, J.A., Larrañaga, P., Inza, I., Bengoetxea, E. (eds) Towards a New Evolutionary Computation. Studies in Fuzziness and Soft Computing, vol 192, pp. 75–102 . Springer, Berlin, Heidelberg (2006). https://doi.org/10.1007/3-540-32494-1_4
2. Li, J., Tan, Y.: Loser-out tournament-based fireworks algorithm for multimodal function optimization. IEEE Trans. Evol. Comput. **22**(5), 679–691 (2017)
3. Li, J., Zheng, S., Tan, Y.: Adaptive fireworks algorithm. In: 2014 IEEE Congress on evolutionary computation (CEC), pp. 3214–3221. IEEE (2014)
4. Li, J., Zheng, S., Tan, Y.: The effect of information utilization: introducing a novel guiding spark in the fireworks algorithm. IEEE Trans. Evol. Comput. **21**(1), 153–166 (2016)
5. Li, Y., Tan, Y.: Enhancing fireworks algorithm in local adaptation and global collaboration. In: Tan, Y., Shi, Y. (eds.) ICSI 2021. LNCS, vol. 12689, pp. 451–465. Springer, Cham (2021). https://doi.org/10.1007/978-3-030-78743-1_41
6. Mohamed, A.W., Hadi, A.A., Agrawal, P., Sallam, K.M., Mohamed, A.K.: Gaining-sharing knowledge based algorithm with adaptive parameters hybrid with imode algorithm for solving cec 2021 benchmark problems. In: 2021 IEEE Congress on Evolutionary Computation (CEC), pp. 841–848. IEEE (2021)
7. Tan, Y., Zhu, Y.: Fireworks algorithm for optimization. In: Tan, Y., Shi, Y., Tan, K.C. (eds.) ICSI 2010. LNCS, vol. 6145, pp. 355–364. Springer, Heidelberg (2010). https://doi.org/10.1007/978-3-642-13495-1_44
8. Zheng, S., Janecek, A., Li, J., Tan, Y.: Dynamic search in fireworks algorithm. In: 2014 IEEE Congress on Evolutionary Computation (CEC), pp. 3222–3229. IEEE (2014)
9. Zheng, S., Janecek, A., Tan, Y.: Enhanced fireworks algorithm. In: 2013 IEEE Congress on Evolutionary Computation (CEC), pp. 2069–2077. IEEE (2013)
10. Zheng, S., Li, J., Janecek, A., Tan, Y.: A cooperative framework for fireworks algorithm. IEEE/ACM Trans. Comput. Biol. Bioinform. **14**(1), 27–41 (2015)